MATHEMATICS 6

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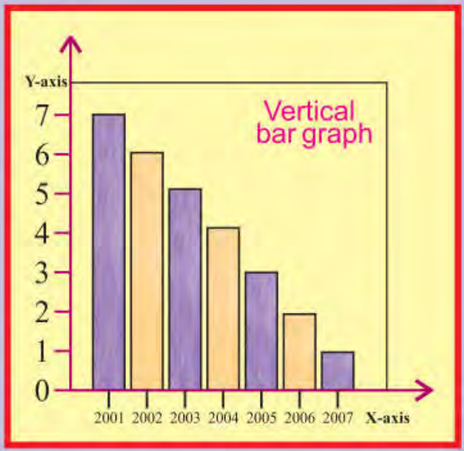

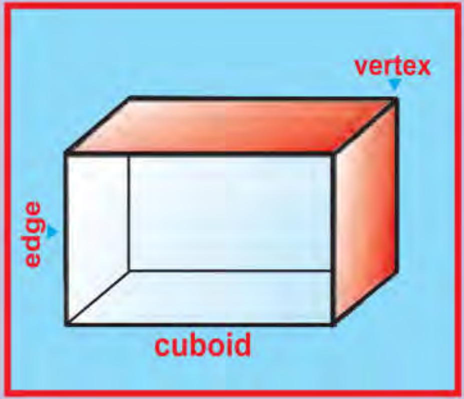

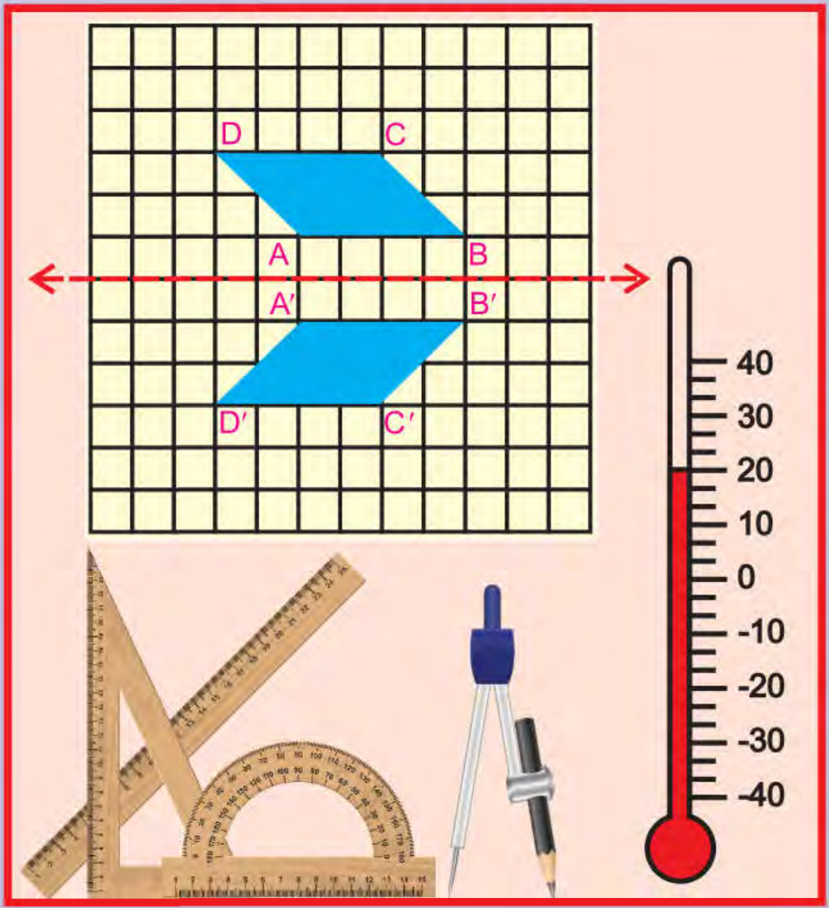

Punjab Curriculum and Textbook Board, Lahore

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(In the Name of Allah the most Beneficent the most Merciful)

MATHEMATICS

6

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PUNJAB CURRICULUM AND TEXTBOOK BOARD LAHORE

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Experimental Edition

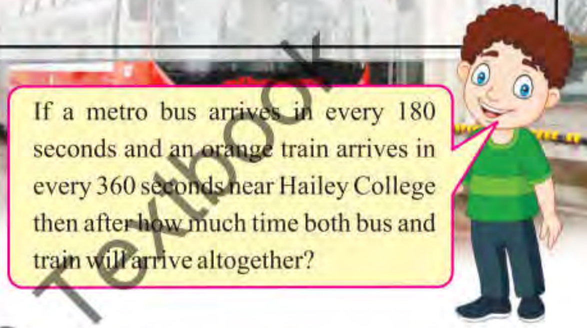
Sub-Domain 1

FACTORS AND MULTIPLES

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Recognize factors of numbers up to 3-digit
- Multiples of numbers up to 2-digit.
- HCF and LCM.



1.1 FACTORS AND MULTIPLES

Factors

"Factors are the numbers which we multiply to get another number say, product or total".

For example, Or 6 (factor) × 3 (factor) = 18 (product)

1.1.1 Defining a Factor

If we divide a dividend by a divisor, we get quotient and remainder. Now look at the given expressions.

$1\overline{)12}$ = 12 (12 − 1 = 2, 2 − 2 = 0, remainder 0), $2\overline{)12}$ = 6 (12 − 12 = 0), $3\overline{)12}$ = 4 (12 − 12 = 0), $4\overline{)12}$ = 3 (12 − 12 = 0), $6\overline{)12}$ = 2 (12 − 12 = 0), $12\overline{)12}$ = 1 (12 − 12 = 0)

12 ÷ 1	= 12
12 ÷ 2	= 6
12 ÷ 3	= 4
12 ÷ 4	= 3
12 ÷ 6	= 2
12 ÷ 12	= 1

From the above examples, it can be seen that the number 12 is completely divisible by 1, 2, 3, 4, 6 and 12. The remainder in each case is zero (0). We can say that the factors of 12 are 1, 2, 3, 4, 6 and 12.

So, *"A factor is a number which divides the dividend completely leaving no remainder".*

Example 1 Which of the given numbers are the factors of 42. **(i)** 3 **(ii)** 5

Solution

(i) 3

$3\overline{)42}$ = 14; 42 − 3 = 12; 12 − 12 = 0

or 42 ÷ 3 = 14 with remainder '0'

As, dividend = quotient × divisor + remainder

42 = 14 × 3 + 0

Since, remainder is zero (0) so, 14 is factor of 42.

(ii) 5

$5\overline{)42}$ = 8; 42 − 40 = 2

or 42 ÷ 5 = 8 with remainder '2'

As, dividend = quotient × divisor + remainder

42 = 8 × 5 + 2

Since, remainder is not zero (0) so, 5 is not factor of 42.

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Fun Fact

- All even numbers will have number 2 as their factor.
- All numbers end with 5 will have 5 as their factor.
- All numbers greater than 0 and end with a '0' will have 2, 5 and 10 as their factors.
- Factors are always whole numbers or integers.

Example 2 Write all the factors of 36 and 40.

Solution

$36 = 1 \times 36$ $36 = 2 \times 18$ $36 = 3 \times 12$ $36 = 4 \times 9$ $36 = 6 \times 6$ As, 1, 2, 3, 4, 6, 9, 12, 18 and 36 can divide 36 completely. Hence, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.	$40 = 1 \times 40$ $40 = 2 \times 20$ $40 = 4 \times 10$ $40 = 5 \times 8$ As, 1, 2, 4, 5, 8, 10, 20 and 40 can divide 40 completely. Hence, the factors of 40 are 1, 2, 4, 5, 8, 10, 20, and 40.

Practice -1 List all the factors of each number. **i.** 84 **ii.** 120 **iii.** 230 **iv.** 352

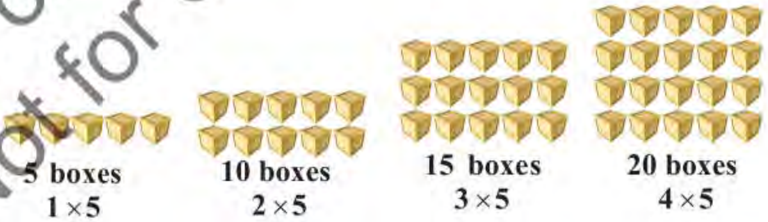
Note Every number greater than 1 has at least two factors. The number 1 and itself.

Remember Stop finding factors when the numbers start repeating.

Multiples

"A multiple is the product or total which we get when we multiply one number by another number."

For example, multiples of 5 when we multiply 5 by another number.



1.1.2 Defining a Multiple

We get multiples of 5 when we multiply it by another number, e.g, 5, 10, 15, 20. Let us consider the following.

$$\begin{array}{r} 8 \\ 2\overline{)16} \\ \underline{16} \\ 0 \end{array} \qquad \begin{array}{r} 6 \\ 3\overline{)18} \\ \underline{18} \\ 0 \end{array}$$

So, $16 \div 2 = 8$ So, $18 \div 3 = 6$

Dividend ÷ Divisor = Quotient

Here, the divisor is called the factor because the remainder is zero (0). 2 is a factor of 16 and 3 is a factor of 18. In both cases the remainder is '0'.

Hence, "multiple of a given number is the dividend, for which the given number is its factor".

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Fun Fact

- A number has unlimited multiples.
- Every number is a multiple of itself.
- Every multiple of a given number is greater than or equal to the given number.

Thinking Time

i. Consider the following facts.

Operation	Result
Even + Even	Even
Even + Odd	Odd
Odd + Even	Odd
Odd + Odd	Even

What will happen when we use the operation subtraction instead of addition?

ii. Consider the following facts.

Operation	Result
Even × Even	Even
Even × Odd	Even
Odd × Even	Even
Odd × Odd	Odd

What will happen when we use the operation division instead of multiplication?

Example 3 Which of the following number is the multiple of 6.

(i) 42 (ii) 52 (iii) 70

Solution

(i) 42

$$\begin{array}{r} 7 \\ 6\overline{)42} \\ 42 \\ \hline 0 \end{array}$$

So, 42 is multiple of 6 because 6 divides 42 completely.

(ii) 52

$$\begin{array}{r} 8 \\ 6\overline{)52} \\ 48 \\ \hline 4 \end{array}$$

52 is not the multiple of 6 because remainder is 4.

(iii) 70

$$\begin{array}{r} 11 \\ 6\overline{)70} \\ 6 \\ \hline 10 \\ 6 \\ \hline 4 \end{array}$$

70 is not the multiple of 6 because remainder is 4.

Example 4 List first six multiples of 7, 9 and 12.

Solution

Multiples of 7	Multiples of 9	Multiples of 12
1 × 7 = 7	1 × 9 = 9	1 × 12 = 12
2 × 7 = 14	2 × 9 = 18	2 × 12 = 24
3 × 7 = 21	3 × 9 = 27	3 × 12 = 36
4 × 7 = 28	4 × 9 = 36	4 × 12 = 48
5 × 7 = 35	5 × 9 = 45	5 × 12 = 60
6 × 7 = 42	6 × 9 = 54	6 × 12 = 72
Hence, 7, 14, 21, 28, 35 and 42 are the first six multiples of 7.	Hence, 9, 18, 27, 36, 45 and 54 are the first six multiples of 9.	Hence, 12, 24, 36, 48, 60 and 72 are the first six multiples of 12.

Practice -2 List the first five multiples of each number. i. 3 ii. 11 iii. 29 iv. 73

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Activity -1

Bring a matchbox and make few multiples of 3 using matchsticks.

Remember

To find the multiples of any given number simply multiply the given number by 1, 2, 3, 4, 5,...

1.2 IDENTIFICATION OF FACTORS UP TO 3-DIGIT NUMBERS

Example 5 Which of the following numbers are the factors of 30 and why?

(i) 5 (ii) 7 (iii) 3 (iv) 11

Solution

(i) 5 is a factor of 30 because it divides 30 completely with zero remainder.
(ii) 7 is not a factor of 30 because it does not divide 30 completely with zero remainder.
(iii) 3 is a factor of 30 because it divides 30 completely with zero remainder.
(iv) 11 is not a factor of 30 because it does not divide 30 completely with zero remainder.

Example 6 Which of the following numbers are the factors of 625 and why?

(i) 25 (ii) 17

Solution

(i) 25 is a factor of 625 because it divides 625 completely with zero remainder.
(ii) 17 is not a factor of 625 because it does not divide 625 completely with zero remainder.

1.3 IDENTIFICATION OF MULTIPLES OF NUMBERS UP TO 4-DIGIT

Example 7 Identify which of the following numbers are the multiples of 13?

(i) 39 (ii) 45 (iii) 65 (iv) 99

Solution

(i) 39 is a multiple of 13. (ii) 45 is not a multiple of 13.
(iii) 65 is a multiple of 13. (iv) 99 is not a multiple of 13.

Practice -3 Write at least 5 multiples and all factors of i. 36 ii. 45 iii. 76

1.4 PRIME NUMBERS AND COMPOSITE NUMBERS

We have learnt about the factors of the numbers in previous section.

Prime numbers

"A natural number which has only two different factors, 1 *and the number itself, is called prime number."*

For example:

(a)	(b)	(c)	(d)
1 × 2	1 × 3	1 × 5	1 × 7

As, 2, 3, 5, 7, … have only two factors so, they are prime numbers. The set of prime numbers is denoted by the capital letter P.

P = {2, 3, 5, …}

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Challenge Numbers 17 and 71 are the prime numbers with reversed digits. Write another pair of prime numbers with reversed digits.

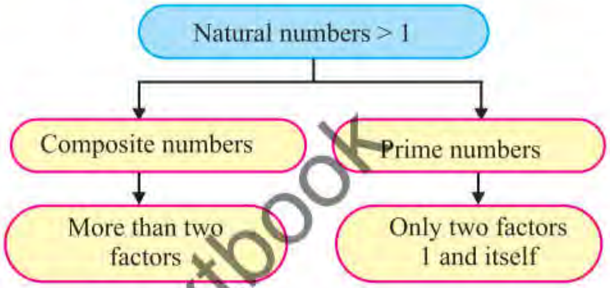
Example 8 Identify the prime number. **(i)** 41 **(ii)** 38 **(iii)** 51

Solution

(i) 41 is a prime number because it has only two factors 1 and 41.

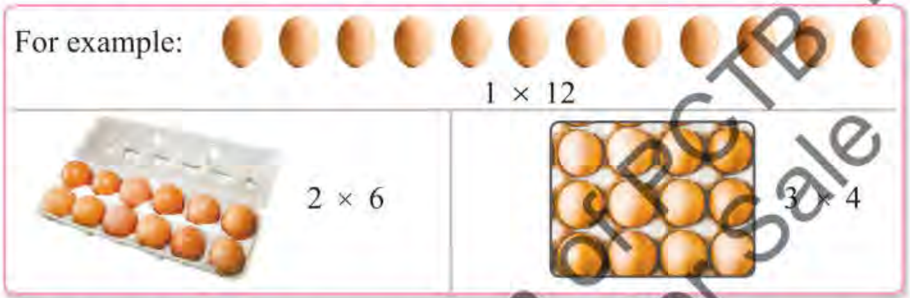
(ii) 38 is not a prime number because it has more than two factors i.e., 1, 2, 19 and 38.

(iii) 51 is not a prime number because it has more than two factors i.e., 1, 3, 17 and 51.



Composite numbers

"A natural number which has more than two different factors is a composite number."



Interesting Information

A composite number can be expressed as a sum of two prime numbers .e.g.

$3 + 5 = 8$

1, 2, 3, 4, 6 and 12 are the factors of 12. Number 12 has more than two different factors. So, 12 is a composite number. Similarly 4, 6, 8, 9, ... have more than two factors so they are composite numbers. The set of composite numbers is denoted by the capital letter C.

$$C = \{4, 6, 8, 9, \ldots\}$$

Example 9 Identify the composite numbers: **(i)** 29 **(ii)** 39 **(iii)** 56

Solution

(i) 29 is not a composite number because it has only two factors.

(ii) 39 is a composite number because it has more than two factors i.e., 1, 3, 13 and 39.

(iii) 56 is a composite number because it has more than two factors i.e., 1, 2, 4, 7, 8, 14, 28 and 56.

Teaching Point

- Ask students to stand in an open space at least an-arm away from each other.
- Tell them that you will be calling out numbers (such as 13, 21 or 101) and they will need to decide if the number is prime or composite.
- If prime they should sit down, if composite they should stand up.

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Example 10 Identify the following numbers as prime, composite and neither prime nor composite numbers.

65, 77, 29, 1, 19, 63, 37

Solution

Prime	Composite	Neither Prime Nor-Composite
19, 29, 37	63, 65, 77	1

Activity -1 Do a magic game. Copy all the numbers 1-100 and then perform with following steps.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Encircle the square box of number 1.
- Leave square box of number 2, and shade all multiples of 2.
- Leave square box of 3, and shade all multiples of 3.
- Leave square box of 5, and shade all the multiples of 5.
- Leave square box of 7, and shade all the multiples of 7.
- All the numbers left shaded are prime numbers, between 1-100.
- At the end list down all the prime numbers between 1-100.

The above process of finding all the prime numbers (1–100) is called the sieve of Eratosthenes.

History

Eratosthenes was a Greek Mathematician. He introduced the sieve of Eratosthenes, an efficient method of identifying prime number. To know more about him visit the below link.

https://www.britannica.com/biography/Eratosthenes

(276BCE-194BCE)

Practice -4 Twin primes are prime numbers that differ by 2. Such as 3 and 5. List all the twin primes between 1-100.

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Exercise - 1.1

1. Write all the factors of the following numbers:

(i) 30 (ii) 41 (iii) 65 (iv) 78 (v) 105 (vi) 118 (vii) 165 (viii) 220

2. Write the first five multiples of the following numbers:

(i) 7 (ii) 13 (iii) 19 (iv) 43 (v) 59

3. Write all prime numbers between 30 and 65.

4. Write all even prime numbers between 50 and 100.

5. List down all even prime natural and whole numbers.

6. List down all odd composite numbers between 50 and 100.

7. Write all neither prime nor composite natural and whole numbers.

Project - 1 Read the Sunday newspaper at your home and write the different digit numbers even, odd, prime, composite and neither prime nor composite numbers. Then write all factors and three multiples of each number.

(i) ________ (ii) ________ (iii) ________ (iv) ________
(v) ________ (vi) ________ (vii) ________ (viii) ________

Thinking Time Which prime number cannot be doubled from the numbers in the 1-10 group?

1.5 PRIME FACTORS OF UP TO 4-DIGIT NUMBERS AND HOW TO EXPRESS THEM IN INDEX NOTATION

In the previous section 1.1 we have learnt how to find factors of a number.

"The process of writing a given number into its factors is known as factorization"

For example, $18 = 3 \times 6$

Here, 3 and 6 are the factors of 18.

Also $18 = 2 \times 9$

Here, again, numbers 2 and 9 are called factors of 18.

Remember

Any composite number can be written as a product of all of its prime factors.

Go Online

Practice the prime factorization by playing online game at http://www.mathplayground.com/factortrees.html

Prime factors

A composite number can be expressed as the product of two or more prime numbers, which are called prime factors.

"The process of factorizing a number into its prime factors is known as prime factorization."

Prime factorization can be done by using two methods.

(i) Division Method. (ii) Factor Tree Method.

Practice -5

Find the prime factors of

i. 30 ii. 210 iii. 633 iv. 310

Example 11 Find the prime factorization of 36

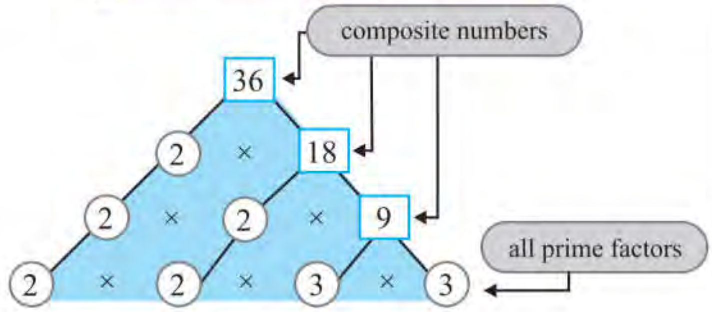
Solution

Division Method

2	36
2	18
3	9
3	3
	1

So, $36 = 2 \times 2 \times 3 \times 3$

Factor Tree Method



So, $36 = 2 \times 2 \times 3 \times 3$

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Index Notation

We are familiar that; $100 = 10 \times 10 = 10^2$

$1000 = 10 \times 10 \times 10 = 10^3$ and $10000 = 10 \times 10 \times 10 \times 10 = 10^4$

The power (index or exponent) of a number says how many times to use the number in factorization.

For example, $36 = 2 \times 2 \times 3 \times 3$

Here, factor 2 is 2 times $= 2^2$ (Index = 2), factor 3 is 2 times $= 3^2$ (Index = 2)

So, prime factorization can be expressed as $36 = 2^2 \times 3^2$

"The representation of prime factors in the exponential form is called index notation".

Now, look at the following example;

The process to factorize a given number and to express its prime factors in the index notation is explained in this example.

Remember

Index

2^5

Base

Example 12 Find prime factorization of 108 then express the result in index notation.

Solution

Division Method

$108 = 2 \times 2 \times 3 \times 3 \times 3$

$= 2^2 \times 3^3$

2	108
2	54
3	27
3	9
3	3
	1

Remember

$2 \times 2 \times 3 \times 3 \times 3$ is expanded form and $2^2 \times 3^3$ is index notation.

Challenge

- I am the product of two different prime factors
- One of my factors is raised to the 2nd power.
- My value is between 62 and 82.
- Who am I?

Note Students should use only one method to find prime factorization. If it is not specified in the problem.

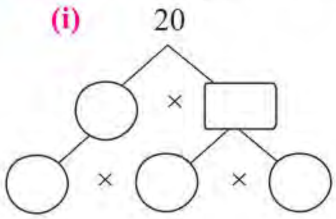
Practice -6 Find the prime factors of the following numbers and express them in index notation.

i. 96 ii. 192 iii. 576 iv. 4,032

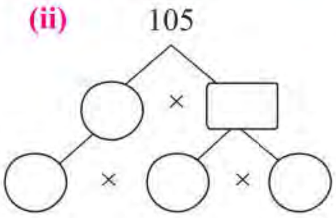
Exercise - 1.2

1. Fill the spaces to complete the factor tree:

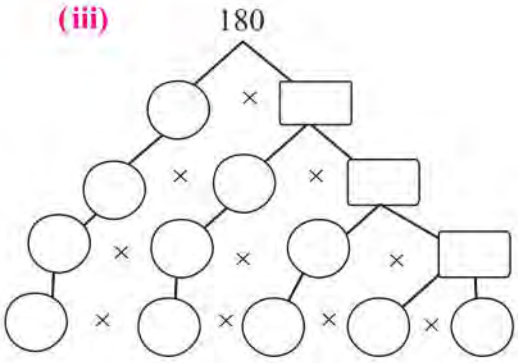
(i)



(ii)



(iii)



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2. Write the following expanded forms into index notation:

(i) $5 \times 5 \times 5 \times 5 \times 7$ **(ii)** $2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$

(iii) $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$ **(iv)** $11 \times 11 \times 11 \times 11 \times 13 \times 19 \times 19 \times 19$

3. Find the prime factors for the following using division method then express in index notation:

(i) 32 **(ii)** 90 **(iii)** 110 **(iv)** 115 **(v)** 225

(vi) 540 **(vii)** 1386 **(viii)** 2205 **(ix)** 6615 **(x)** 6075

4. Find the prime factors of the following numbers using factor tree method then express in index notation:

(i) 64 **(ii)** 125 **(iii)** 100 **(iv)** 500 **(v)** 188

(vi) 648 **(vii)** 1512 **(viii)** 3240 **(ix)** 7560 **(x)** 7875

5. Prove that prime factors of 2,600 are same using division method and factor tree method.

6. I am a product of three different numbers.
My greatest prime factor is 13.
I am between 100 and 200.
Who am I?

7. Find the number that is the product of the most prime numbers below 100.

8. Write these index notations into expanded form:

(i) $2^4 \times 3^2 \times 5^3$ **(ii)** $3^5 \times 5^4 \times 11^3$ **(iii)** $5^6 \times 7^3 \times 11^5$ **(iv)** $2^7 \times 3^4 \times 13^3 \times 17^2$

Project - 2 Search a 2-digit number, where the square of the sum of its digits is equal to the number obtained when its digits are reversed.

1.6 H.C.F AND L.C.M OF TWO OR THREE NUMBERS UP TO 3-DIGIT NUMBERS

1.6.1 HCF (Highest Common Factor)

We have learnt little more about factors and H.C.F (highest common factors) in our previous classes.

"H.C.F of two or more numbers is a greatest or highest number which divides all the given numbers."

The H.C.F is also called the greatest common factor (G.C.F) or greatest common divisor (G.C.D).

The above definition of H.C.F can also be defined as:

"H.C.F or highest common factor of two or more given numbers is the common factor of given numbers which is the greatest of all common factors".

Remember The H.C.F is always smaller than or equal to the smallest number in the set of given numbers.

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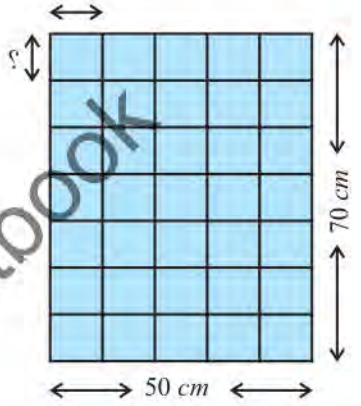
Now, look at the following example.

Example 13 Yesterday Alisha plans to cover a blue sheet of paper completely with identical square boxes. The sheet of paper is 70 *cm* long and 50 *cm* wide. How can she find the side of the largest possible square?

Solution Length of blue sheet of paper = 70 *cm*

Breadth of blue sheet of paper = 50 *cm*



We should divide each side into equal groups with equal lengths. This is same as to find the factors of 50 and 70.

Factors of 70 = 1, 2, 5, 7, 10, 14, 35, 70

Factors of 50 = 1, 2, 5, 10, 25, 50

The common factors of 70 and 50 are 1, 2, 5 and 10.

As, the Highest Common Factor of 50 and 70 is 10.

Hence, the side of the largest possible square is 10 *cm*.

H.C.F of two or more than two numbers up to 3-digit by Prime Factorization Method

To find H.C.F using prime factorization, simply use the following rule.

H.C.F = Product of common prime factors of all given numbers.

Example 14 Find H.C.F of 27, 54 and 315 by using prime factorization.

Solution First use division to find the prime factors of each number.

3	27
3	9
3	3
	1

So, 27 = 3 × 3 × 3

2	54
3	27
3	9
3	3
	1

So, 54 = 2 × 3 × 3 × 3

3	315
3	105
5	35
7	7
	1

So, 27 = 3 × 3 × 5 × 7

common to 27, 54, and 63

Remember

H.C.F (Highest Common Factor) is also called G.C.D (Greatest Common Divisor).

Hence, H.C.F = 3 × 3 = 9 i.e. H.C.F of 27, 54 and 315 is 9.

H.C.F of two or more than two numbers up to 3-digit by Long Division Method

To find H.C.F using long division method we use the following rules.

- Divide the largest number by the smallest number.
- Take remainder as divisor and divide the first divisor by it.
- Now, remainder will be taken as divisor and second divisor will be treated as dividend.

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- Continue this process again and again until remainder becomes zero.
- The last divisor will be H.C.F.
- In case of three numbers, first we find H.C.F of any two numbers. Then continue this process with remaining number till the last divisor. The last divisor is H.C.F of all the three numbers.

Example 15 Shumaila has a box of 45 orange flavoured sweets and Nadeem has a box of 110 mango flavoured sweets. They have to divide up the sweets into small plates with equal number of sweets, each plate containing either orange flavoured or mango flavoured sweets only. Find the possible number of sweets in each plate.

Solution Orange flavoured sweets = 45
Mango flavoured sweets = 110

To find the possible equal number of sweets in each plate, we should find the highest common factor for 45 and 110.

3	45
3	15
5	5
	1

2	110
5	55
11	11
	1

As, $45 = 3 \times 3 \times 5$
$110 = 2 \times 5 \times 11$
So, H.C.F = 5
Hence, each plate will contain 5 sweets.

Example 16 Nadeem wants to cut two wires into pieces of equal length. If the total length of wires are 108 *m* and 144 *m* respectively then what is the maximum length of each piece?

Solution To find the maximum length of each piece we will find H.C.F of 108 and 144.

```
108) 144 (1
     108
     ----
      36) 108 (3
          108
          ----
            0
```

As, H.C.F of 108 and 144 is 36.
Hence, Nadeem will cut the wires into equal pieces with maximum length of each piece is 36 *m*.

Remember If the H.C.F of two or more numbers is 1, then the numbers are called co-prime.

Example 17 Find H.C.F of 65, 91 and 117 using long division method.

Solution **Step-I** First we find H.C.F of 91 and 117.

```
91) 117 (1
     91
    ----
    26) 91 (3
        78
       ----
       13) 26 (2
           26
          ----
            0
```

So, H.C.F of 91 and 117 is 13.

Step-II

Now, we find H.C.F of 13 and 65.

```
13) 65 (5
    65
   ----
    0
```

Hence,
The H.C.F of 65, 91 and 117 is 13.

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Practice -7

Find the greatest length of measuring tape which can be used to measure exactly 160 *m*, 280 *m* and 200 *m* length.

Exercise - 1.3

1. Find H.C.F of each set of numbers using prime factorization method:

(i) 6 and 9 **(ii)** 25 and 45 **(iii)** 18 and 28

(iv) 5, 30 and 60 **(v)** 14, 28 and 112 **(vi)** 36, 54 and 108

2. Find H.C.F of each set of numbers using long division method:

(i) 110 and 170 **(ii)** 104 and 234 **(iii)** 400 and 3996

(vi) 32, 96 and 256 **(v)** 30, 168 and 288 **(vi)** 80, 215, 245 and 720

3. What is H.C.F of any two prime numbers?

4. Can the highest common factor of 8 and 64 be greater than 8? Explain your reasoning.

5. Aleezey has 16 Science books, 28 Mathematics books and 30 English books. She wants to pack these books equally without mixing the books and with no left over. What is the greatest number of books she can put in each package?

6. There are 50 students of grade VI, 70 students of grade VII and 80 students of grade VIII. A teacher wishes to arrange them in equal number of students in each row. Find the greatest number of students that could be arranged in a row.

7. A room is 10 *m* 15 *cm* long and 8 *m* 80 *cm* wide. The floor of the room is to be paved with square tiles. Find the length of the largest tile that can be used.

Activity -2

- Write your 3-digit pocket money on a sheet of paper.
- Find H.C.F of three numbers you write on the paper.

1.6.2 L.C.M (Least Common Multiple)

We have learnt about common factors and L.C.M (Least Common Multiple) in previous grades.

"*L.C.M of two or more numbers is a least number which is divisible by all the given numbers*".

The above definition of L.C.M can also be defined as:

"L.C.M or least common multiple of two or more numbers is the smallest of all their common multiples".

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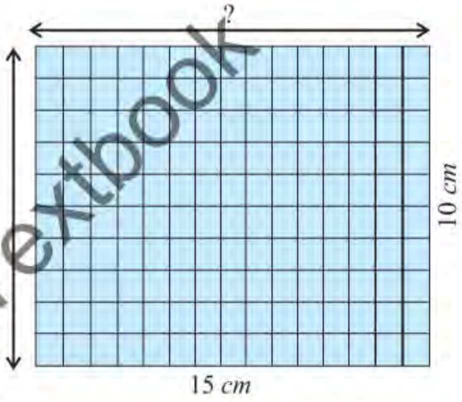
Now, look at the following example.

Example 18 One Sunday Nuzat plans to draw 15 *cm* long and 10 *cm* wide rectangle on a plastic sheet. She then makes copies of rectangle to form a square. Find the length of each side of this square.

Solution Length of small rectangle = 15 *cm*

Breadth of small rectangle = 10 *cm*

First, we must find the multiples of 10 and 15 to get the length of a side of square.



Multiples of 10 = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, 260, 270, 280, 290, 300, 310, …

Multiples of 15 = 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195, 210, 225, 240, 255, 270, 285, 300, 315, …

The first two common multiples of 10 and 15 are 30 and 60.

The smallest is 30. This is called L.C.M (Least Common Multiple).

Hence, the length of each side of square is 30 *cm*.

Remember
- If one number is a factor of the other then L.C.M is the greater number.
- If the given numbers are prime numbers then L.C.M is the product of these numbers.

L.C.M of two or more numbers by Prime Factorization Method

To find L.C.M by using the prime factorization method we use the following rule.

L.C.M = Product of common two or more prime factors × Product of non-common two or more prime factors.

Example 19 Find the L.C.M (Least Common Multiple) of 54 and 144.

Solution

2	54
3	27
3	9
3	3
	1

2	144
2	72
2	36
2	18
3	9
3	3
	1

Here, $54 = 2 \times 3 \times 3 \times 3$

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Common prime factors = 2, 3, 3

Non-common prime factors = 2, 2, 2, 3

So, L.C.M = Product of common prime factors × Product of non-common prime factors

$= 2 \times 3 \times 3 \times 2 \times 2 \times 2 \times 3$

$= 18 \times 24 = 432$

Remember

If three or more than three numbers are given to find the L.C.M, then consider a factor as common factor if it is common factor of two numbers or more than two numbers.

Challenge

Rida packs boxes of mangoes and boxes of oranges. Each box contains the same number of fruits. Rida packs 64 mangoes and 56 oranges.
Nosheen says, 'There will be 7 pieces of fruit in each box.'
Noureen says, 'There will be 8 pieces of fruit in each box.'
Who is correct? Explain your answer.

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L.C.M of two or more numbers up to 3-digit by Division Method

To find L.C.M of two or more numbers by prime factorization needs a lots of calculation. So, the simple method is known as division method.

Step – I Write all the given numbers in a horizontal line, separate them by commas.

Step – II Choose a smallest possible prime number, which exactly divides at least one of the given numbers.

Step – III Write quotient directly under the number in the next row. If any number not divided exactly, then bring it down in the next row.

Step – IV Again choose a smallest possible prime number which exactly divides at least one of the numbers in second row.

Step – V Continue this process until all co-prime numbers are left in the last row.

Step – VI At the end multiply all the prime numbers by which we have divided and the co-prime numbers left in the last row. i.e. (L.C.M = Product of all divisors).

Thinking Time What is the smallest 4-digit number which is divisible by 21, 56 and 63?

Example 20 Minahil says to her friend Emaan that she is thinking of a number. Which is completely divisible by 16, 24 and 36 respectively. What secret number that Minahil could be thinking of ?

Solution The secret number can be found by using L.C.M.

First given number = 16

Second given number = 24

Third given number = 36

L.C.M = product of all divisors

= 2 × 2 × 2 × 2 × 3 × 3 = 144

Hence, Minahil can think 144.

2	16 ,	24 ,	36
2	8 ,	12 ,	18
2	4 ,	6 ,	9
2	2 ,	3 ,	9
3	1 ,	3 ,	9
3	1 ,	1 ,	3
	1 ,	1 ,	1

Practice -8 Waqas is planting trees. He has enough trees to plant 7, 9 and 15 trees in each row. What is the least number of trees Waqas could have?

Exercise - 1.4

1. Find L.C.M of each set of numbers using prime factorization method:

(i) 8 and 18 **(ii)** 28 and 49 **(iii)** 20, 90 and 180

(iv) 25, 35 and 45 **(v)** 15, 25 and 65 **(vi)** 150, 250 and 350

2. Find L.C.M of each set of numbers using division method:

(i) 14 and 63 **(ii)** 25 and 45 **(iii)** 14, 28 and 84

(iv) 7, 28 and 98 **(v)** 220, 240, 260 and 280

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3. A frog and a grasshopper start jumping together and jump along the same path. The frog always jumps 25 *cm* and the grasshopper always jumps 15 *cm*. At what distance they will meet again?

4. A university bus arrives near UET Lahore in every 280 seconds and an orange train arrives in every 360 seconds. How often do train and bus arrive at the same time?

5. I am thinking of a number that is divisible by 29, 58 and 116. What is the smallest possible number that I could be thinking of ?

6. The traffic lights at GPO chowk changes after every 30 seconds, 60 seconds, 90 seconds and 120 seconds respectively. If they change simultaneously at 9 a.m after what time will they change again simultaneously? (**Hint:** 1 minute = 60 seconds).

7. Find the smallest 3 digit number divisible by 16, 24 and 30 respectively.

1.7 RELATIONSHIP BETWEEN HCF AND LCM

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We are familiar that "The H.C.F is the greatest factor of two or more than two numbers which divides the number exactly with no remainder", while "The L.C.M of two or more than two numbers is the smallest number which is divisible by given numbers exactly". After discussing the definition of H.C.F and L.C.M, we will focus on the relationship between H.C.F and L.C.M along with real life problems.

Example 21 Find the relationship between H.C.F and L.C.M of 12 and 21.

Solution First, we find the highest common factor (H.C.F.) of 12 and 21 is 3.
Then we find the least common multiple (L.C.M.) of 12 and 21 is 84.
H.C.F. × L.C.M. = 3 × 84 = 252
Also, the product of numbers = 12 × 21 = 252
Therefore, product of H.C.F. and L.C.M. of 12 and 21 = product of 12 and 21.

Example 22 Find the relationship between H.C.F and L.C.M of 32 and 48.

Solution Again, let us consider the two numbers 32 and 48

Prime factors of 32 and 48 are:

$32 = 2 \times 2 \times 2 \times 2 \times 2$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
L.C.M. of 32 and 48 is 96, H.C.F. of 32 and 48 is 16;
L.C.M. × H.C.F. = 96 × 16 = 1536
Product of numbers = 32 × 48 = 1536

So, from the explanation given in the above examples we conclude that the product of H.C.F and L.C.M of two numbers is equal to the product of two numbers.

or, H.C.F. × L.C.M. = First number × Second number or, $\text{L.C.M.} = \dfrac{\text{First number} \times \text{Second number}}{\text{H.C.F.}}$

Challenge Highest common factor (H.C.F) and least common multiple (L.C.M) of two numbers are 10 and 399 respectively. One number is 190, find the other number.

Exercise - 1.5

1. Find the relationship between H.C.F and L.C.M of the following numbers:

(i) 10 and 25 **(ii)** 8 and 28 **(iii)** 34 and 68

(iv) 35 and 77 **(v)** 110 and 230 **(vi)** 160 and 380

2. Find the relationship between H.C.F and L.C.M of 680 and 584.

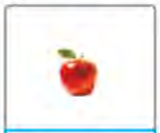
3. The H.C.F of two numbers is 36 and their L.C.M is 1260. If one of the numbers is 72, find the other number.

4. The H.C.F of two numbers is 48 and their L.C.M is 1440. If one of the numbers is 240, find the other number.

Activity -4 Find the relationship between the H.C.F and L.C.M of the prices of your Science and Mathematics textbooks.

1.8 SQUARE OF NUMBERS UP TO 2-DIGIT NUMBERS

Observe the following arrangements of apples.


1-apple

4-apples

9-apples

16-apples

But 8-apples or 10-apples cannot be arranged in this way. Numbers or number of objects that can be arranged into a square array are known as square numbers.

Note A square box/array should be full if we need to count the number as a square number. Here 12-apples are arranged in a square, but it does not full a square. So, 12 is not a square number.

12-apples

Example 23 Find the square number of the integers –3 and 7.

Solution To find the square of given integers or numbers, we simply multiply the number by itself. So,

$(-3) \times (-3) = (-3)^2 = 9$ and $7 \times 7 = 7^2 = 49$

Where, 9 and 49 are square numbers.

Activity -4

Colour the square numbers in red on the given number chart.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

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Exercise - 1.6

1. Identify the square numbers in the following numbers:

(i) 18 **(ii)** 25 **(iii)** 17 **(iv)** 17 **(v)** 17
(vi) 144 **(vii)** 79 **(viii)** 169 **(ix)** 68 **(x)** 121

2. Calculate the squares of the following numbers:

(i) 13 **(ii)** –8 **(iii)** 23 **(iv)** 17 **(v)** –31
(vi) 43 **(vii)** –3 **(viii)** 57 **(ix)** –71 **(x)** 93

Project - 3 Write the ages of all your family members on a chart paper. Recognise and identify whose age is a square number. Also, calculate the square of all ages.

Review Exercise 1

1. Each of the following question is followed by four suggested options. In each case select the correct option.

(i) A number which divides the dividend completely is called:
a. factors b. multiples c. L.C.M d. H.C.F

(ii) Zero (0) is multiple of every number except:
a. one b. two c. itself d. five

(iii) Any given number is even if the digit on its ones place is multiple of:
a. 1 b. 2 c. 3 d. 4

(iv) Any given number is an odd number if the digit on its ones place is not the multiple of:
a. 1 b. 2 c. 3 d. 7

(v) A composite number can always be expressed as a ________ of two primes:
a. product b. difference c. sum d. none of these

(vi) A number neither prime nor composite is __________
a. 1 b. 2 c. 3 d. 4

(vii) The product of factors of a given number is always equal to:
a. prime number b. composite number
c. given number d. even number

(viii) In 5^4
a. 4 is base and 5 is power b. 5 and 4 are bases
c. 4 and 5 are powers d. 5 is base and 4 is power

(ix) A number which divides all the given numbers is called:
a. H.C.F b. L.C.M c. Prime d. Composite

(x) A number which is divisible by all the given numbers is called:
a. H.C.F b. L.C.M c. Prime d. Composite

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2. **(i)** Define factors and give an example.
(ii) Define Multiples and give an example.
(iii) What are prime numbers?
(iv) What are composite numbers?
(v) What do you know about index notation?
(vi) Define prime factorization.
(vii) Define H.C.F (Highest Common Factor).
(viii) Define L.C.M (Least Common Multiple).

3. For an admission test the number of participants in grade VI, grade VII and grade VIII are 60, 84 and 108. Find the minimum number of participants who can sit in a row.

4. Three local taxies A, B and C arrive at a stop. Taxi A arrives at the stop every 20 minutes, taxi B arrives every 30 minutes and taxi C arrives every 40 minutes. All three taxies arrive at the stop at 10:30 am altogether. At what time will the three taxies arrive at the stop again?

5. Find the largest number which when divided by 24, 36 and 60 leaves 3 as remainder in each case.

6. Three bells toll at regular interval of 10 minutes, 20 minutes and 30 minutes respectively. If they toll altogether at 12:00 noon. At what time will they next toll together?

Go Online Practice the highest common factor (H.C.F) and least common multiples (L.C.M) by playing games at: https://www.transum.org/software/sw/starter_of_the_day/students/H.C.F.L.C.M.asp
https://www.transum.org/software/sw/starter_of_the_day/students/H.C.F.L.C.M.asp?level=2

Summary

- A factor is a number which divides the dividend completely leaving no remainder.
- Multiple of a given number is the dividend, for which the given number is its factor.
- A natural number has only two different factors 1 and the number itself, is called a prime number.
- A number which has more than two different factors, is called a composite number.
- The process of factorizing a number into its prime factors is know as prime factorization.
- H.C.F of two or more numbers is a highest number which divides all the given numbers.
- L.C.M is a least number which is divisible by all the given numbers.

Activity -5

Match the square numbers by colouring the block with same colour.

4^2	225	25	8^2	49
100	3^2	7^2	169	15^2
12^2	81	5^2	16	144
64	13^2	9	10^2	9^2

Teaching Point The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

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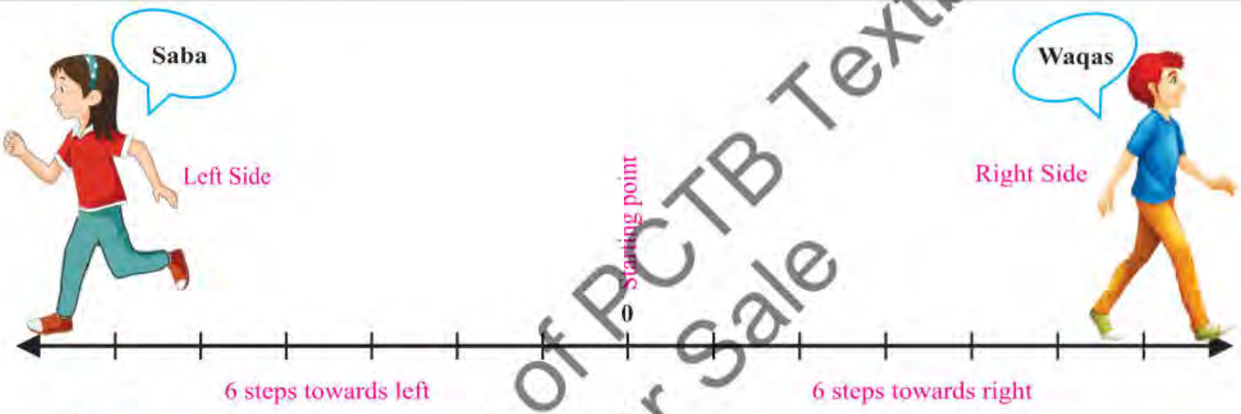
Sub-Domain 2

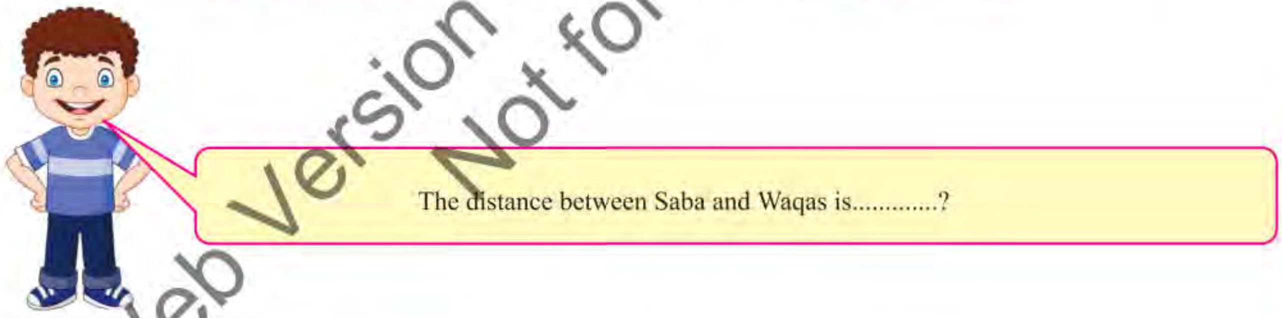
INTEGERS

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Recognize and identify integers (positive integers, negative integers and neutral integer).
- Calculate absolute or numerical value of an integer.
- Using a number line, compare and arrange a given list of integers and their absolute values in ascending and descending order.





2.1 INTEGERS

We know about the natural numbers and whole numbers. Also we have learnt about the number line of natural numbers and whole numbers respectively.

"Integers are in the form of positive numbers, negative numbers and zero (0) too."

The symbol we use for the set of integers is Z because of Zahlen, Zahlen is a German word for integers.

Teaching Point Tell the students about perfect numbers. The numbers which are equal to the sum of its positive divisors are known as perfect numbers. i.e $6 = 1 + 2 + 3$. So, 6 is a perfect number.

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Remember To understand the integers. Simply keep the following figures representation in your mind.

Zero	Natural numbers	Whole numbers	Negative numbers	Integers

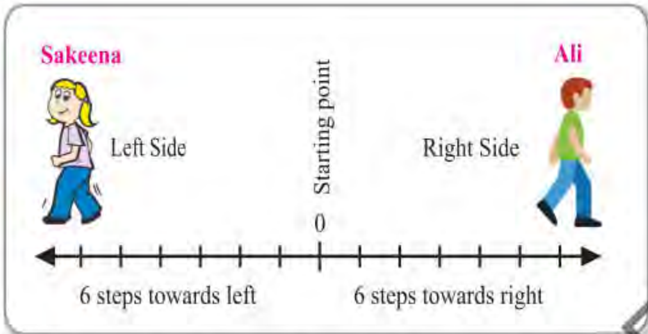
So, the collection of integers can be understood by the given diagram.

Integers

2.2.1 Identification of Integers

Integers are also known as directed numbers because these are used to represent distance ..., –3, –2, –1, 0, 1, 2, 3, ... along with the position or direction.

For example, Sakeena and Ali walk away from the starting point but they walk in opposite direction.



History Arbermouth Holst was a German Mathematician. He introduced integer in the year 1563. He spent 15 years to invent a number system of addition and multiplication. Later on in 1890, Japanes Mathematicians worked on Arbermouth Holst's number system and created 'integers'. To know more about him, visit the link. https://mathematicacademy. weeebly.com/history-of-math-connection.html

If Ali walked 6 steps heading towards right and Sakeena walked 6 steps heading towards left. The distance of 6 steps on the right side from starting point is shown by +6. The distance of 6 steps on the left side from starting point is shown by –6.

In our real life we use positive numbers as well as negative numbers, the concept of negative numbers is derived from our real life situation, for example:

The temperature recorded 25°C above zero as positive and correspondingly –25°C below zero as negative temperature.

Similarly, Shahbaz earned Rs. 1500 in a day. He spent Rs. 500. We can write these numbers as integers.

Earned Rs. 1500, +1500 rupees ← *Read: "Positive 1500 rupees"*

Spent Rs. 500, –500 rupees ← *Read: "Negative 500 rupees"*

So, *"The natural numbers are 1, 2, 3, ... are also called positive integers and the corresponding numbers –1, –2, –3, ... are called negative integers".*

We observed besides positive and negative integers, there is an integer which is neither positive nor-negative that is zero (0).

So, *"0 is an integer which is neither positive nor negative".* It is called a neutral integer.

Activity -1 Think and list any three real life situations where you use negative and positive numbers.

(i). ------------------------- **(ii).** ------------------------- **(iii).** -------------------------

Teaching Point Ask students to give real life scenarios where they have seen negative numbers. Also, show following video to discuss real life examples of integers. (https://www.youtube.com/watch?v=9w7gwFA1HNI)

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Note Zero (0) is greater than every negative integer and smaller than every positive integer.

Example 1 Identify the integers in the following numbers.

$$-42, +35, +21.6, -15.9, +\frac{18}{23}, +12, +29.8, -1\frac{5}{8}$$

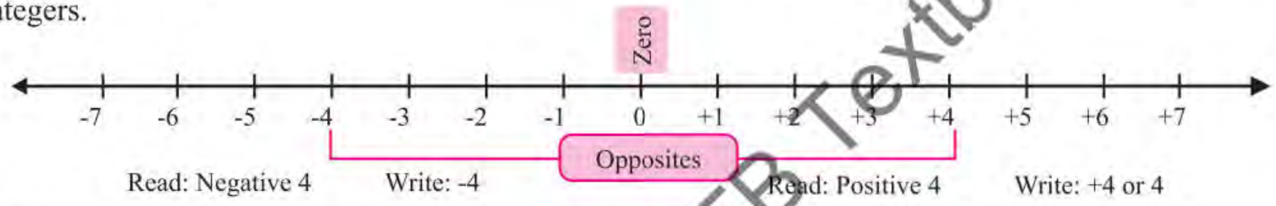
Solution Integers: $-42, 35, 12$

Non-integers: $+21.6, -15.9, +\frac{18}{23}, +29.8, -1\frac{5}{8}$

Challenge
There is only one number spelled with the same number of letters as itself.
Which number is this?

2.2.2 Representation of Integers on Number Line

Integers can be represented on number line. The number line can help us to see the relationship between integers.

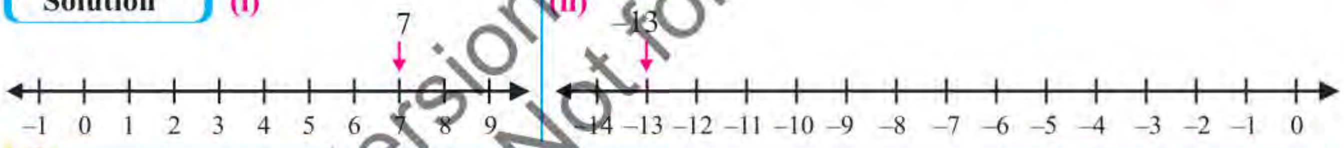


Two integers are opposite (additive inverse) if they are at the same distance from zero on the number line, but on opposite sides of zero. Each integer has an opposite. For example.

(i) The opposite integer of $+4$ is -4. **(ii)** The opposite integer of -4 is $+4$.

Example 2 Represent the following numbers on the number line. **(i)** 7 **(ii)** -13

Solution **(i)** **(ii)**



Remember The distance between any two points on the number line is always same or equal.

Practice -1 Represent the following numbers on the number line. **(i)** -8 **(ii)** 11 **(iii)** -21

Exercise - 2.1

1. List the natural numbers between 4 and 11.
2. What is a smallest natural number?
3. Write all the whole numbers less than 10.
4. Write all the natural number less than 13.
5. Give the short answer for each of the following question:

(i) Is every prime number a natural number?
(ii) Write the difference between natural numbers and whole numbers.
(iii) How many natural numbers are the whole numbers?

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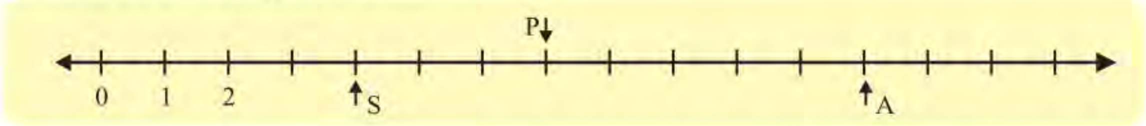
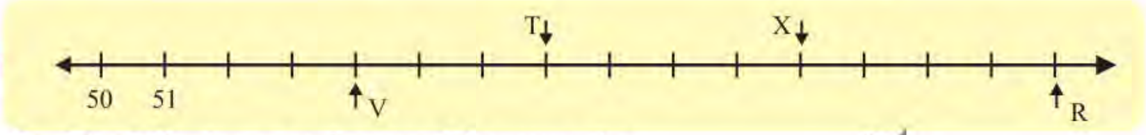
6. On what number do all the alphabet stands for?

(i)



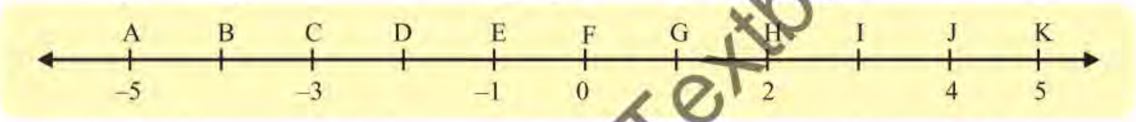
(ii)



7. Write each of the following as an integer with correct sign.

(i) 48 degrees cooler. (ii) 48 degrees warmer.

8. Identify the corresponding point to the each integer given on the following number line:



(i) B (ii) D (iii) G (iv) –5 (v) –3 (vi) 0

9. Draw number line and represent the following points on it.

(i) 0, –8, 6, 3, –11 and –13 (ii) 3, 8, 5, –11, –5 and –7

10. Write the opposite for the following integers.

(i) 14 (ii) –24 (iii) 65 (iv) –39 (v) 0 (vi) –420

11. Identify the integers in the following numbers.

$$+12, -21.4, -17, 1\frac{2}{5}, 3\frac{9}{11}, -11.25, 18.4, +26, -57, 0$$

12. Hadi has Rs. 12 in his pocket, but Neelam gives him Rs. 4 and Hamza gives him Rs. 3. How much money Hadi has in all? Show all this information on number line.

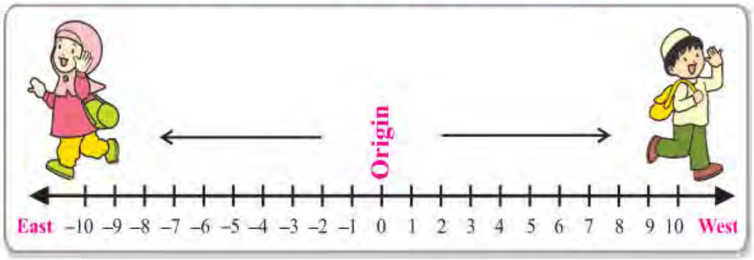
13. Describe and show your position on number line, if you begin at 8, move right 8 steps and then move left 8 steps.

14. When Saba woke up, her temperature was 101°F. Two hours later it was 4°F lower. What was her temperature then?

Project - 1 Prepare a chart paper for the number line. Represent your obtained marks of grade-5 final exam on the number line.

2.3 ABSOLUTE OR NUMERICAL VALUE OF AN INTEGER

The concept of absolute value has many uses. We know about the number line. Every integer on the number line represents a distance from zero.



For example, Fatima is moving 10 steps to east and Shahid is moving 10 steps to west

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from the origin on the number line. Fatima is standing at point –10 and Shahid is standing at point +10, but cover the same distance from the origin.

So, we can say that

"Absolute value or numerical value of a number is its distance from zero on the number line and it is always positive or zero."

It is always denoted by symbol '| |'. In the above example distance 10*m* left from zero can be written as '|–10|' and read it as absolute value of –10 is 10.

2.3.1 Arrangement of absolute values of the given integers in ascending or descending order

Look at the following examples to understand the concept of ascending and descending order of absolute values.

Example 3 Arrange the absolute values of the following numbers in ascending order.

7, –14, 0, 15, –12, –2, 11

Solution 7, –14, 0, 15, –12, –2, 11

Here, |7| = 7, |–14| = 14, |0| = 0, |15| = 15, |–12| = 12, |–2| = 2 and |11| = 11

So, the required arrangements in ascending order of absolute values is 0, 2, 7, 11, 12, 14, 15.

Example 4 Arrange the absolute value of the following numbers in descending order.

–6, 3, –13, 5, 0, –2, 12, –18

Solution –6, 3, –13, 5, 0, –2, 12, –18

Here, |–6| = 6, |3| = 3, |–13| = 13, |5| = 5, |0| = 0, |–2| = 2, |12| = 12 and |–18| = 18

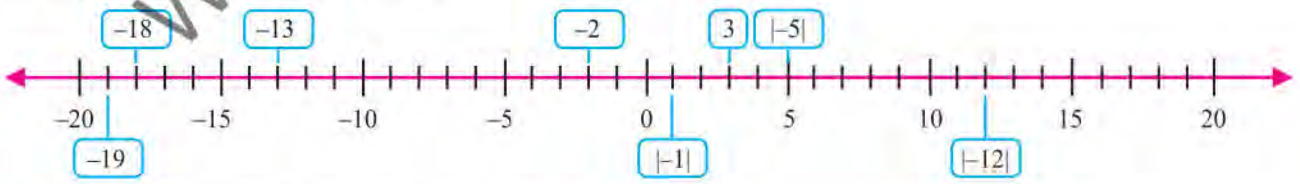
So, the required arrangement in descending order absolute values is 18, 13, 12, 6, 5, 3, 2, 0.

Example 5 Use number line to arrange the following numbers in ascending order:

–2, –13, –18, |–5|, 3, –19, |–1|, |–12|

Solution Since, |–5| = 5 and |–1| = 1, |–12| = 12

Now , use number line to order them



Hence, Ascending order: –19, –18, –13, –2, 1, 3, 5, 12

Teaching Point Use the following link to show the video for the explanation of the concept of absolute value. You can pause/stop the video at several interval and ask the questions from the students.

http://www.youtube.com/watch?v=zxaT8Arckjo

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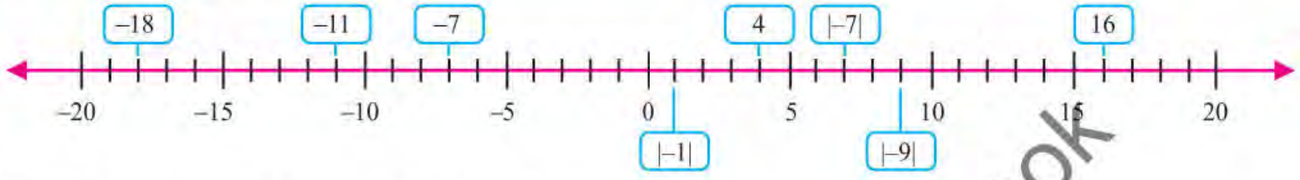
Example 6 Use number line to arrange the following numbers in descending order.

|–1|, –11, 4, |–9|, –7, –18, |–7|, 16

Solution Since, |–1| = 1, |–9| = 9, |–7| = 7

Now , use number line to order them



Hence, descending order: 16, 9, 7, 4, 1, –7, –11, –18

Go Online Use the following links to practice of arranging integers.

- https://www.mathplayground.com/mobile/numberballs_fullscreen.htm
- https://www.math-play.com/Millionaire-Game-Absolute-value/Millionaire_Game-Absolute-Value_htm/5.html

Exercise - 2.2

1. Arrange the following integers in ascending order:

(i) –7, 0, –4, 4, 3, –2, 1 **(ii)** –1, –6, 7, –3, 0, 6, 17

2. Arrange the following integers in descending order:

(i) –4, –2, 7, 16, 0, –1 **(ii)** 13, 24, –10, 0, 8, 5

3. Write the absolute or numerical value of the following integers:

(i) –45 **(ii)** 116 **(iii)** –209 **(iv)** 0 **(v)** –2813 **(vi)** –501

4. Arrange the absolute values of the following integers in ascending order:

(i) –13, 0, –4, 12, 16, –9, 17 **(ii)** –18, 14, –12, –84, 44, 48

5. Arrange the absolute values of the following integers in descending order:

(i) 34, –9, 12, 17, –16, –21 **(ii)** 109, –215, 0, –85, –141, 149, 215

Project - 2 Make a booklet and explain

- Which number is greater, –17 or –30?
- Prepare five flash cards for different integers and arrange them in ascending and descending order using number line.

Review Exercise 2

1 Each of the question or incomplete statement is followed by four suggested options. In each case select the one that is correct.

(i) The set of integers is mostly denoted by English capital letter:

a. W b. N c. Q d. Z

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(ii) Integers are consisted of:

a. Only positive whole numbers b. Only negative whole numbers
c. Positive and negative whole number d. Positive and negative whole number with zero

(iii) An integer which is neither positive nor negative is:

a. 0 b. 1 c. 2 d. 3

(iv) On number line the distance between any two integers is always:

a. Equal b. Doubled c. Triple d. Half

(v) The opposite numbers of +2 and –3 are:

a. +2 and 3 b. –2 and 3 c. –2 and –3 d. 2 and –3

(vi) The absolute value of a number is its distance from:

a. –1 b. 0 c. 1 d. 2

(vii) The numerical value of –345 is:

a. 345 b. –345 c. 34 d. 3405

(viii) The ascending order of 1, –1, 0, 2, –3, 5 is:

a. –3, –1, 0, 1, 2, 5 b. 0, –1, 1, 2, –3, 5 c. 5, 2, 1, 0, –1, –3 d. –1, 1, 0, –2, 3, 5

(ix) The descending order of 1, –1, 0, 2, –3, 5 is:

a. –3, –1, 0, 1, 2, 5 b. 0, 1, –1, 2, –3, 5 c. 5, 2, 1, 0, –1, –3 d. –1, 0, 1, 2, –3, 5

(x) $-|-9| =$ ________:

a. –9 b. 9 c. 10 d. –10

(xi) $|-10| =$ ________:

a. –10 b. 10 c. –9 d. 9

2. Define the following:

(i) Natural numbers (ii) Whole numbers (iii) Integers (iv) Absolute value

3. Represent the following integers on the number line:

(i) –7, 8, –2, –1 (ii) 3, –2, 0, –4 (iii) –5, –1, –3, 0 (iv) 7, –8, –10, 0, –13, 6

4. Arrange the following in ascending order:

(i) –3, 4, –4, 0, 2, 7 (ii) –7, 0, 4, –11, 1, –13 (iii) 15, –4, 4, 5, –6, 0

5. Arrange the following in descending order:

(i) –4, 2, 0, –5, –7 (ii) –5, 12, 0, –3, 5 (iii) –6, 5, –2, 0, 8, 3

6. Represent the following absolute values on the number line:

$-|8|, |-11|, -|-4|, |3|, -|7|, -3$

7. An elevator is on the eighth floor. It goes 10 floors down and then up 3 floors. At which floor is the elevator now?

8. It is –3°C tonight. Neelam predicts it will be 18°C warmer by noon tomorrow. What will be the temperature by noon tomorrow?

9. Calculate your age from your birth year, and explain which arithmetic operation you need to find your age.

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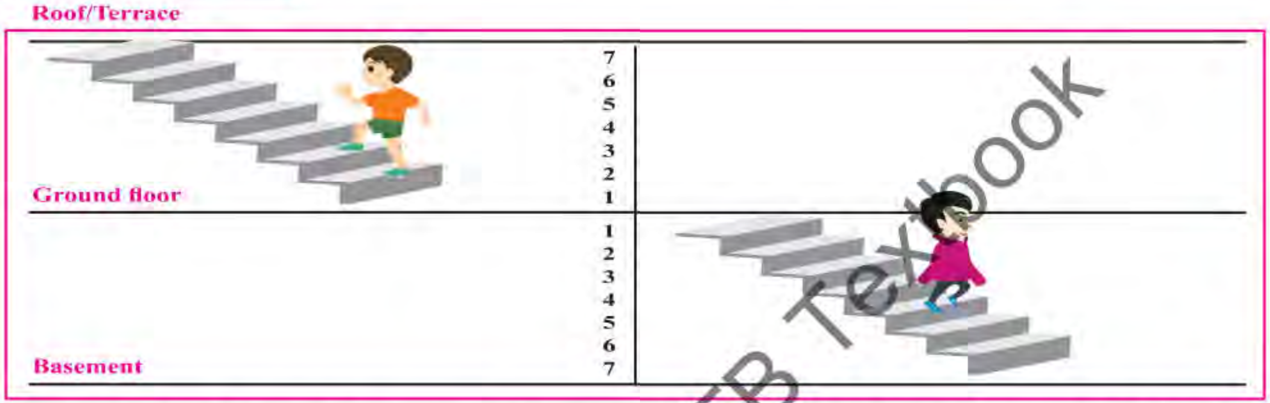
10. Write the absolute value for each integer: 15, 0, –18, –15, 13, 215, –131, 54

11. Two metro buses left Kalma Chowk Station on the same time. One travelled 13 *km* towards Gujjumata. While the other travelled 9 *km* towards Shahdra, in the opposite direction. Find the total distance between the two buses.

Activity -2



If you move 2 steps up towards roof/terrace and your friend moves 5 steps down towards basement of a building then what is the difference between you and your friend's position? Can you represent both positions on number line? If yes then do it.

Summary

- Natural numbers are counting number. The set of natural numbers is denoted by capital letter of English alphabet 'N'. e.g, N = {1, 2, 3, 4, 5, ...}
- The number zero (0) together with the natural or counting number gives us the list of whole numbers. The set of whole numbers is denoted by capital letter of English alphabet 'W'. e.g, W = {0, 1, 2, 3, 4, ...}.
- Positive integers are numbers greater than zero (0), lie to the right of zero (0) on the number line.
- Negative integers are numbers less than zero (0), lie to the left of zero (0) on a number line.
- Set of integer is {0, ±1, ±2, ±3, ...}.
- Zero (0) is an integer which is neither positive nor negative.
- Any number on number line lying to the left of zero (0) is negative.
- Any number on the number line lying to the right of zero (0) is positive.
- The positive distance from zero (0) to any number on the number line is called absolute value.
- The absolute value of a number is always positive.

Teaching Point The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

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Sub-Domain 3

LAWS OF INTEGERS

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Add up to 2-digits like and unlike integers.
- Verify commutative and associative laws.
- Subtract up to 2-digits like and unlike integers.
- Multiply up to 2-digits like and unlike integers.
- Verify commutative, associative and distributive laws.
- Divide like and unlike integers up to 2-digits.

Student -1	Student -2
$5 + 3 \times 4 \div 2$ $= 5 + 3 \times 2$ $= 5 + 6$ $= 11$	$5 + 3 \times 4 \div 2$ $= 8 \times 4 \div 2$ $= 32 \div 2$ $= 16$

Who is doing well?

3.1 ADDITION AND SUBTRACTION UP TO 2-DIGITS LIKE AND UNLIKE INTEGERS

A number line can also be used to find the sum or difference between given integers. The addition of integers is same as the addition of whole numbers using number line. But there is a slight difference. On the number line when we add the positive integers we move towards right and when we add the negative integers we move towards left.

Remember

- The integers with the same signs are called like integers.
- The integers with different signs are called unlike integers.

3.1.1 Addition of Integers

Sum of two or more like integers

The procedure to find the sum of two or more given negative integers is in the following examples.

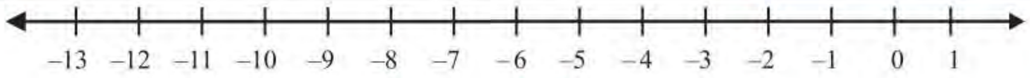
Example 1 Use number line to find the sum of –6 and –7.

Remember

To find the sum, we add the given numbers.

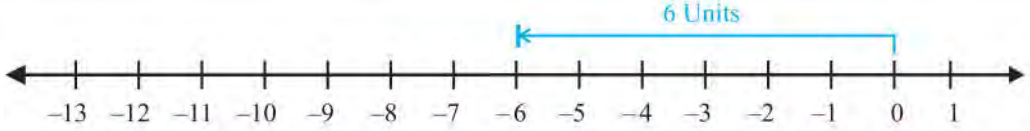
Solution $(-6) + (-7)$

Step-i



Draw a number line.

Step-ii



Consider 0 as starting point and move 6 steps towards left reaching at –6

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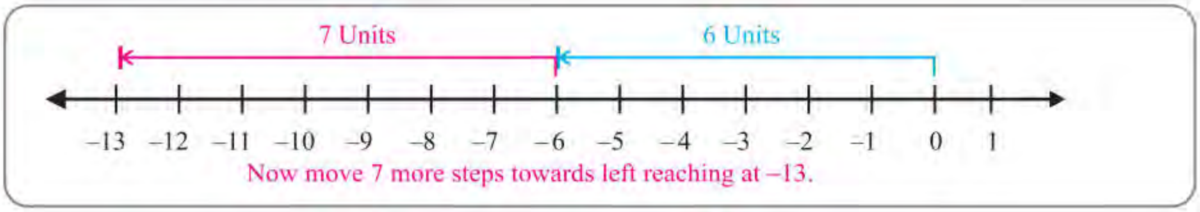
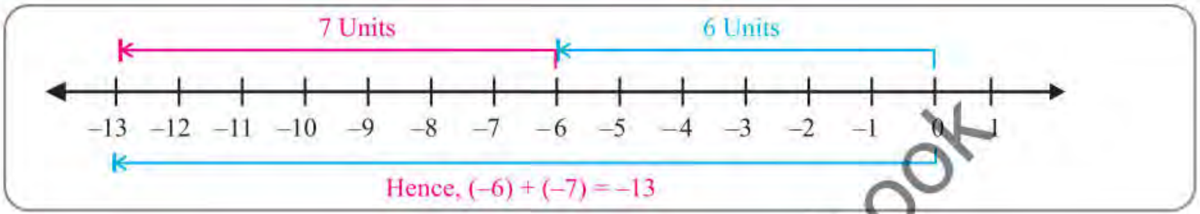
Step-iii

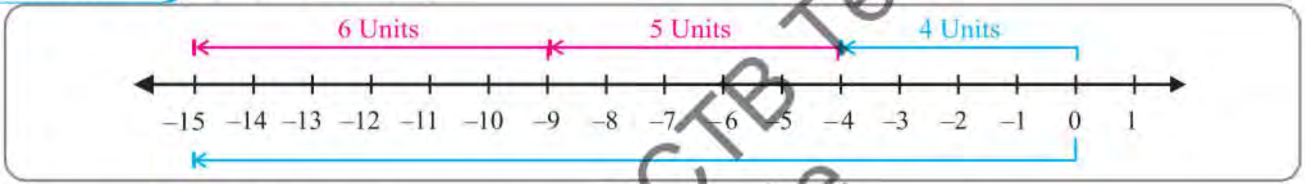


Step-iv



Example 2 Use number line to find the sum of –4, –5, and –6.

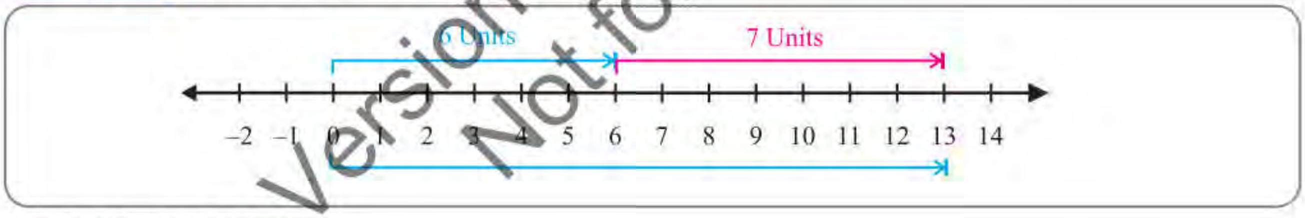
Solution (–4) + (–5) + (–6)



Hence, (–4) + (–5) + (–6) = –15

Example 3 Use number line to find the sum of 6 and 7.

Solution (+6) + (+7)



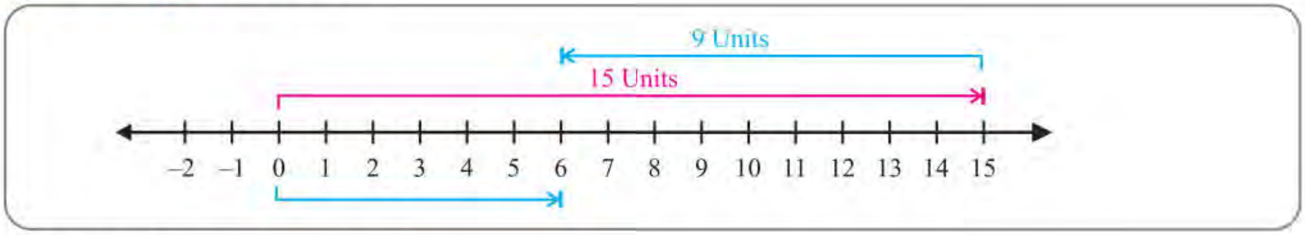
Hence, (+6) + (+7) = +13

Sum of unlike integers

The procedure to find the sum of two given integers is illustrated in the following examples.

Example 4 Use number line to find the sum of 15 and –9.

Solution (+15) + (–9)



Hence, (+15) + (–9) = +15 – 9 = +6

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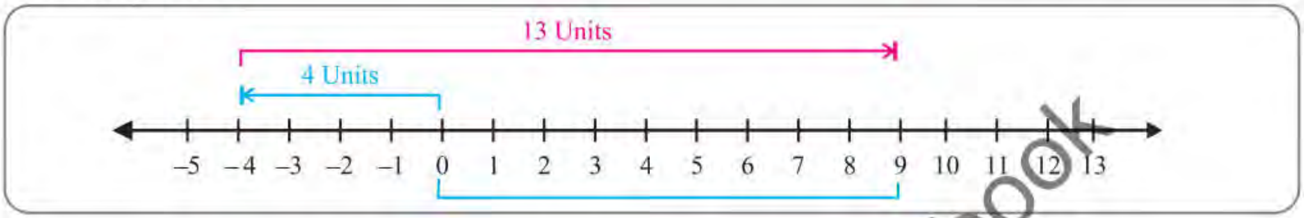
Example 5 Use number line to find the sum of – 4 and 13.

Solution (– 4) + (+13)

Consider "0" as starting point and move 4 steps towards left, and then move 13 steps towards right. Reached at point +9.



Hence, (– 4) + (+13) = – 4 + 13 = +9

Practice -1 Use number line to find the sum of:

(i) 5 and –7 (ii) –11 and –5 (iii) – 8 and 18 (iv) 6 and 10

3.1.2 Subtraction of Integers

Since the subtraction is the inverse process of addition. A number line can be used to find the difference between two integers. The word " difference" is a key word that means subtraction. The same procedure can be used to subtract integers which we have used to add integers using number line.

Recognize subtraction as the inverse process of addition

Subtraction is the inverse process of addition because "subtracting" a positive integer (subtrahends) from another given positive integer is the same as adding a negative integer (subtrahend) with reverse direction in another given positive integer.

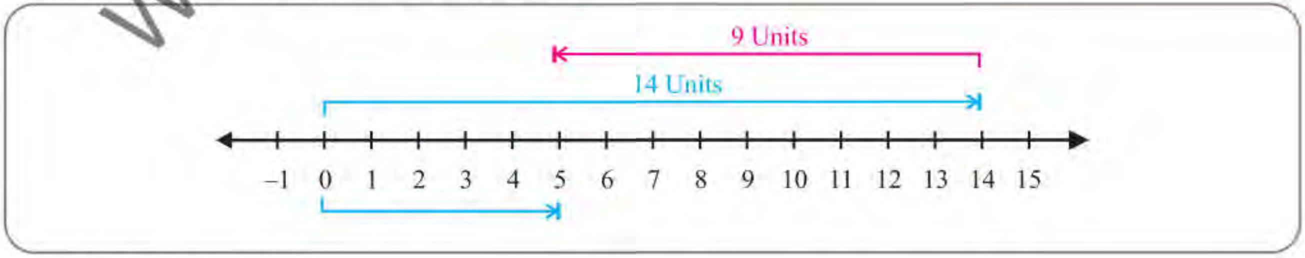
Subtraction of like integers

Example 6 Subtract: (i) 9 from 14 (ii) –15 from –9

Remember
To find the difference, we subtract the given numbers.

Solution (i) (+14) – (+9)

"Subtracting" +9 from 14 is same as "adding" 14 to –9.



Hence, (+14) – (+9) = +5

Teaching Point Use the following links to explain the concept of adding and subtracting integers.

- https://www.youtube.com/watch?v=CfkaifC7tGY
- https://www.youtube.com/watch?v=1DKWG5CBeek

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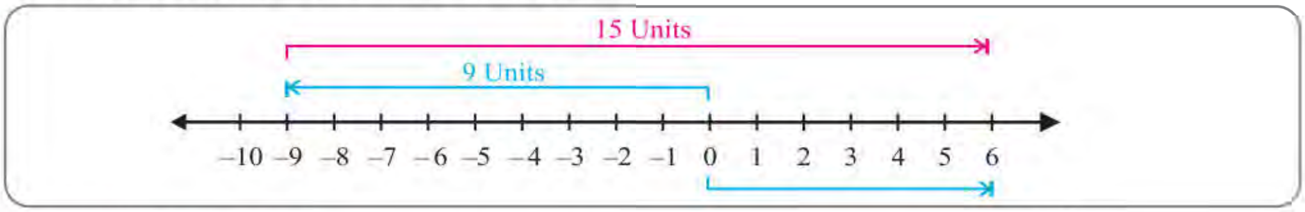
(ii) $(-9)-(-15) = -9+15$

Subtracting (–15) from (–9) is same as adding (–9) to 15.



Hence, $(-9)-(-15) = +6$

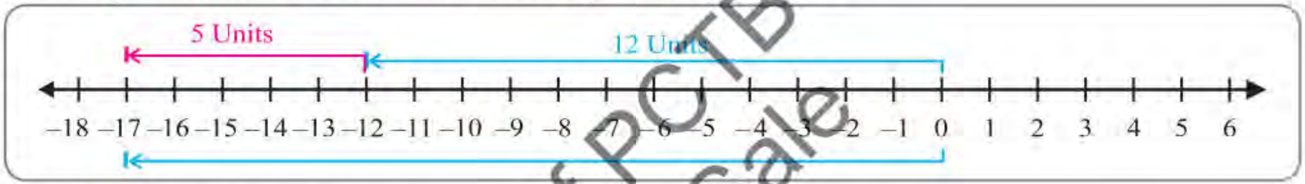
Subtraction of unlike integers

The procedure to subtract the unlike integers is illustrated in the following examples.

Example 7 Subtract: **(i)** +5 from –12 **(ii)** 13 from –5

Solution **(i)** $(-12)-(+5) = -12-5$

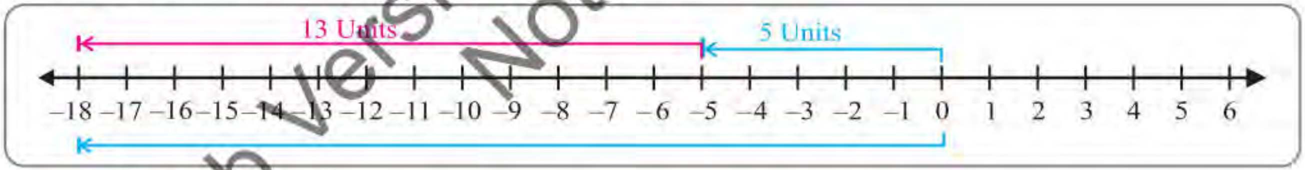
Subtracting +5 from –12 is the same as adding –12 to –5.



Hence, $(-12)-(+5) = -17$

(ii) $(-5)-(+13) = -5-13$

Subtracting (+13) from (–5) is same as adding (–5) to –13.



Hence, $(-5)-(+13) = -18$

Practice -2 Use number line to subtract the following integers.

(i) 8 and 17 **(ii)** –9 and –14 **(iii)** –11 and 7 **(iv)** 8 and –16

3.1.3 Subtraction of one Integer from the other by changing the sign of the Integer

The following rule can be used to subtract integer from another integer.

Rule Change the sign of the integer to be subtracted and add according to the rules of addition.

Remember While doing subtracting always write the greater integer first.

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Look at the following examples.

Example 8 Simplify the following:

(i) $(+9)-(+4)$ **(ii)** $(+16)-(-7)$ **(iii)** $(-40)-(-26)$ **(iv)** $(-68)-(+43)$

Solution

(i)	(ii)	(iii)	(iv)
$(+9)-(+4)$ $=+(9-4)$ $=+5$	$(+16)-(-7)$ $=+(16+7)$ $=+23$	$(-40)-(-26)$ $=-(+40-26)$ $=-14$	$(-68)-(+43)$ $=-68-43$ $=-(68+43)$ $=-111$

Example 9 In a holy month of Ramadan at noon on a Friday, the temperature of Lahore was +41°C. At the time of opening roza the temperature was decreased by 13°C. What was the temperature at the time of opening the roza?

Solution Temperature at noon = +41°C

Decrease in temperature till opening time of roza = +13°C

Now, temperature at opening time of roza = (+41°C) – (+13°C)

= +(41°C – 13°C)

= +28°C

Hence, the temperature at opening time of roza was +28°C.

Practice -3 Subtract the following by changing the signs: **(i)** $(-37)-(+14)$ **(ii)** $(-94)-(-29)$

We can perform four basic mathematical operations (+, –, ×, ÷) on integers. In our previous classes, we have learnt about these basic operations.

3.1.4 Addition and Subtraction of Integers without using number line

We know how to find the sum and difference of integers using number line. In previous section 3.1.1 and 3.1.2 we have discussed about them. Now in this section we will learn about addition and subtraction of two integers without using the number line. Here we will use the simple rule of addition and subtraction.

Addition of two integers with like signs

To add the given integers with like signs simply follow the method illustrated in the following example

Example 10 Find the sum of: **(i)** +14 and +19 **(ii)** –11 and –16

Solution

(i)	(ii)
+14 and +19 $+(14+19)=+33$ As, the common sign is '+' so the sum is positive.	–11 and –16 $-(11+16)=-27$ As, the common sign is '–' so the sum is negative.

Go Online

Use the link to play the online game.
http://www.mathplayground.com/ASB_Spidermatchinteger.html

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Practice -4 Find the sum of: **(i)** 18 and 19 **(ii)** –29 and –57

Addition of two integers with unlike signs

To add the given integers with unlike signs simply follow the method illustrated in the following example:

Example 11 Find the sum of

(i) –16 and +11 **(ii)** +70 and –22 **(iii)** 0 and –70 **(iv)** +31 and 0

Solution

(i) (–16) + (+11)
= –16 + 11
= –5

As, the sign of greater value is negative so, the result is negative.

(ii) (+70) + (–22)
= 70 – 22
= 48

As, the sign of greater value is positive so, the result is positive.

(iii) (0) + (–70)
= 0 – 70
= –70

As, the sign of greater value is negative so, the result is negative.

(iv) (+31) + (0)
= +31 + 0
= +31

As, the sign of greater value is positive so, the result is positive.

Remember While adding two integers with different signs simply subtract the integers and put the sign with result which is greater valued sign.

Note The sum of any integer with zero (0) always give the same integer in result.

Practice -5 Find the sum of: **(i)** 13 and –18 **(ii)** –27 and 48

3.2 VERIFICATION OF COMMUTATIVE AND ASSOCIATIVE LAWS

The following laws of addition are true for integers.

Commutative law of addition

According to this law of addition, the sum of two integers unchanged by changing the "order". For example, 2 + 3 = 3 + 2 ⇒ 5 = 5

Example 12 Verify the commutative law under addition for whole numbers 23 and 27.

Solution By the commutative law of addition.

23 + 27 = 27 + 23

Take L.H.S
= 23 + 27
= 50

Now, take R.H.S
= 27 + 23
= 50

As, L.H.S = R.H.S So, 23 + 27 = 27 + 23

Hence, commutative law of addition is verified.

Remember
Commutative law with respect to subtraction does not hold. e.g.
8 – 3 ≠ 3 – 8

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Associative law of addition

According to this law of addition changing the "grouping" of addends does not change the sum.

For example, $(4 + 5) + 7 = 4 + (5 + 7)$

$9 + 7 = 4 + 12$

$16 = 16$

Did You Know?

The sum of zero (0) and an integer is the same as that integer.

For example, $0 + 7 = 7$ or $7 + 0 = 7$

and the number '0' is called additive identity.

Example 13 Verify the associative law of addition for the integers:

(i) 15, –16 and 19 **(ii)** –14, –18 and 23

Solution

(i) 15, –16 and 19

By associative law

$(15 - 16) + 19 = 15 + (-16 + 19)$

Take L.H.S $= (15 - 16) + 19$

$= -1 + 19 = 18$

Now, take R.H.S $= 15 + (-16 + 19)$

$= 15 + 3 = 18$

As, L.H.S = R.H.S

Hence, associative law of addition verified.

(ii) –14, –18 and 23

By associative law

$(-14 - 18) + 23 = -14 + (-18 + 23)$

Take L.H.S $= (-14 - 18) + 23$

$= -32 + 23 = -9$

Now, take R.H.S $= -14 + (-18 + 23)$

$= -14 + 5 = -9$

As, L.H.S = R.H.S

Hence, associative law of addition verified.

Practice -6

(i) Verify commutative law for the integers –49 and 57.

(ii) Verify associative law for the integers –17, 43 and 64.

Go Online Play the different games online by using the below links.

https://www.mathgames.com/skill/3.37-properties-of-addition

https://www.mathgames.com/skill/3.38-solve-using-properties-of-addition

https://www.mathgames.com/skill/7.96-properties-of-addition-and-multiplication

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Exercise - 3.1

1. Use number line to simplify the following:

(i) $(+7) + (+9)$ **(ii)** $0 + (-10)$ **(iii)** $(-8) + (-6)$

(iv) $(+17) - (+8)$ **(v)** $(+11) + 0$ **(vi)** $(+13) + (+18) + (+8)$

(vii) $(+15) + (-3) + (-5)$ **(viii)** $(-8) - (-9)$ **(ix)** $7 - (-8)$

2. Use number line to find the difference of the following integers:

(i) $(+17) - (+11)$ **(ii)** $(-34) - (+16)$ **(iii)** $(+2) - (-4)$ **(iv)** $(-94) - (-19)$ **(v)** $(-8) - (-3)$

(vi) $(-81) - (+16)$ **(vii)** $(+12) - (+10)$ **(viii)** $(-9) - (-4)$ **(ix)** $47 - 18$

3. Verify associative law of addition for the integers 17, –21 and 25.

4. The sum of two integers is –107. One of them is –49, find the other.

5. Verify the commutative property of addition for 39 and –51.

6. The difference between two integers is 48, one of them is 14, find the other.

7. Two orange trains left Samanabad station at the same time. One travelled 14 *km* towards Ali Town, while the other travelled 10 *km* towards Dera Gujran in the opposite direction. Find the total distance between both trains.

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Project - 1 Measure the temperature of yourself for one week. Then find the difference between the highest and the lowest temperature.

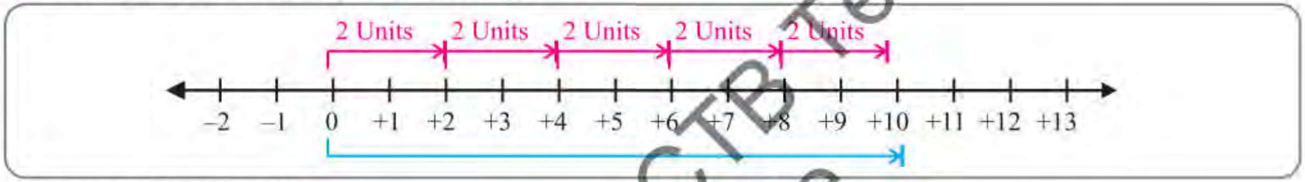
3.4 MULTIPLICATION AND DIVISION OF INTEGERS

We have learnt about the multiplication and division of two or more numbers in our previous classes. Now in this section we will learn multiplication and division of two or more integers.

3.4.1 Multiplication of Integers

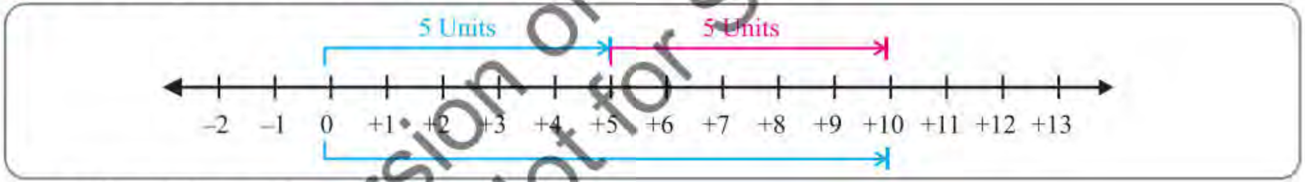
As we are familiar that multiplication is the shortest form of repeated addition.
For example (+5) × (+2). Multiplying (+5) with (+2) mean five jumps, each jump is two steps in the same direction.

(a). Five jumps, each jump is of two steps.



Hence, (+5) × (+2) = +10

(b). Two jumps, each jump is of five steps.



Hence, (+2) × (+5) = +10

Multiplication of two integers with like signs

When we multiply two integers having the "same" signs (both positive or both negative). Then the product of these integers is a positive integer. Now consider the following examples:

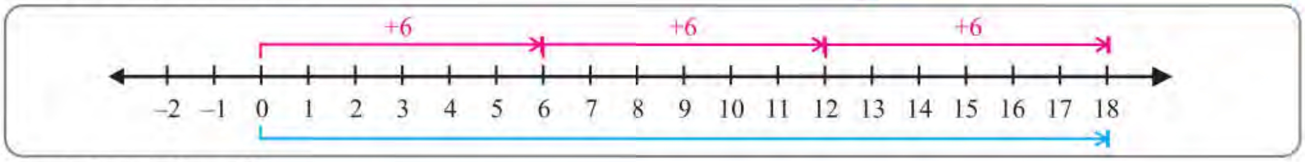
Example 14 Solve the following using number line:

(i) (+3) × (+6) **(ii)** (–6) × (–2)

Solution **(i)** (+3) × (+6)

Multiplying (+3) with (+6) means three jumps, each jump is six steps in the same direction.

Remember
When we multiply even number of integers with same sign the product is always positive.



Hence, (+3) × (+6) = +18

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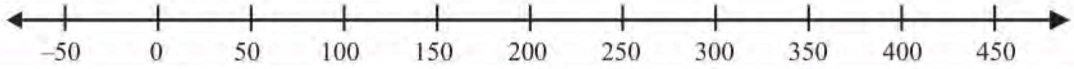
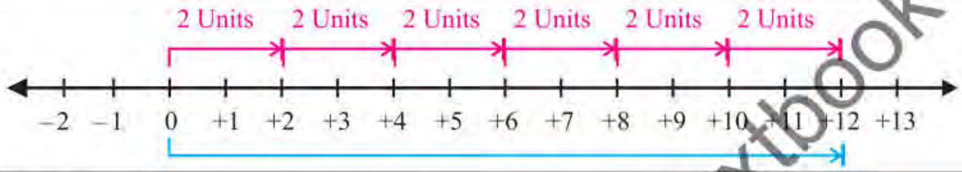
Note In order to draw a large number on the number line, we need to fix a scale on the number line with equal intervals. For example, we take the distance between two points is equal to 50, then

1 unit = 1 × 50 = 50,   7 units = 7 × 50 = 350



(ii) (–6) × (–2)

Multiplying (–6) with (–2) means six jumps, each jump is two steps in opposite direction of –6.



Hence, (–6) × (–2) = +12

Remember

Rules for Multiplication
- (+) × (+) = +
- (+) × (–) = –
- (–) × (+) = –
- (–) × (–) = +

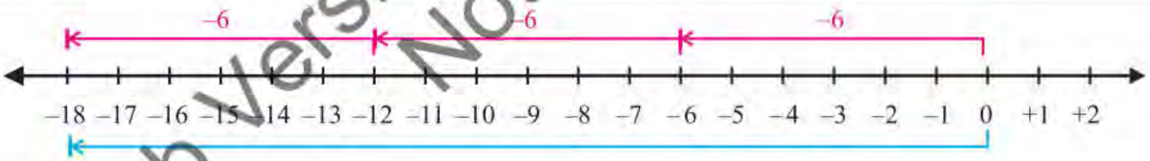
Multiplication of two integers with unlike signs

When we multiply two integers having the "*different*" signs (one positive and other negative), the product of these integers is a negative integer. Now consider the following examples:

Example 15 Solve the following using number line: **(i)** (–3) × (+6) **(ii)** (+5) × (–3)

Solution **(i)** (–3) × (+6)

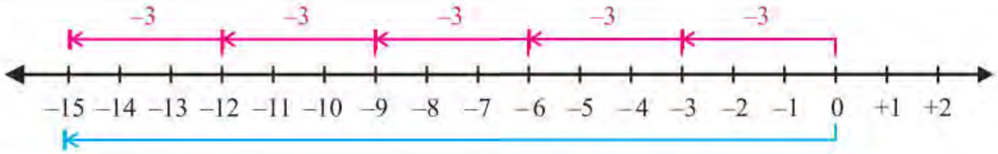
Multiplying (–3) with (+6) means three jumps, each jump is of six steps in the same direction of (–3).



Hence, (–3) × (+6) = –18

(ii) (+5) × (–3)

Multiplying (+5) with (–3) means five jumps, each jump is of three steps in the same direction of (–3).



Hence, (+5) × (–3) = –15

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Example 16 Find the product of the following integers without using the number line:

(i) (+23) and (+22) **(ii)** (–8) and (–15) **(iii)** (+24) and (–10) **(iv)** (–36) and (+11)

Solution

(i) (+23) × (+22)
= + (23 × 22)
= + 506

(ii) (–8) × (–15)
= + (8 × 15)
= + 120

(iii) (+24) × (–10)
= – (24 × 10)
= – 240

(iv) (–36) and (+11)
= – (36 × 11)
= – 396

Practice -6

Multiply:

(i) 10 by –12
(ii) –14 by –9
(iii) 34 by 8
(iv) –16 by 10

3.4.2 Division of Integers

Division of integers is same as the division of whole numbers. The procedure for dividing integers is same as we used for multiplying the integers.

"The division is the inverse process of multiplication."

7 × 6 = 42 42 ÷ 6 = 7
same numbers

7 × 6 = 42 42 ÷ 7 = 6
same numbers

Division of two integers with like signs

Since the division is an inverse process of multiplication, the division of an integer (divided) by another given integer (divisor) means to find an integer (quotient) which when multiplied by the given integer, division will give dividend.

Look at the following expression:

(+63) ÷ (+9) = (+7)
Dividend ÷ Divisor = Quotient

Now, consider the following example:

Example 17 Workout: **(i)** (+45) ÷ (+5) **(ii)** (–54) ÷ (–6)

Solution

(i) (+45) ÷ (+5)

$= (+45) \times \left(+\frac{1}{5}\right)$

$= +\left(\frac{45}{5}\right)$ ∵ (+) × (+) = (+)

= + 9

(ii) (–54) ÷ (–6)

$= (-54) \times \left(-\frac{1}{6}\right)$

$= +\left(\frac{54}{6}\right)$ ∵ (–) × (–) = (+)

= + 9

Practice -6 Divide. **(i)** –48 ÷ 6 **(ii)** 130 ÷ (–5) **(iii)** –216 ÷ (–4) **(iv)** 105 ÷ 7

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Division of two integers with unlike signs

Now, look at the expression:

(+70)	÷	(–10)	=	(–7)
↓		↓		↓
Dividend	÷	Divisor	=	Quotient

To divide (+70) by (–10) means to find an integer such as (–7) which when multiplied by (–10) will give (+70). i.e.

$$(-10) \times (-7) = (+70)$$

Example 18 Solve **(i)** (+195) ÷ (–5) **(ii)** (–80) ÷ (+8)

Solution

(i) (+195) ÷ (–5)

$= (+195)\times\left(-\frac{1}{5}\right) \quad \because (+)\times(-)=(-)$

$= -\left(\frac{195}{5}\right)$

$= -39$

(ii) (–80) ÷ (+8)

$= (-80)\times\left(+\frac{1}{8}\right) \quad \because (-)\times(+)=(-)$

$= -\left(\frac{80}{8}\right)$

$= -10$

Division of an integer by zero (0)

Division of any number by zero (0) never comes to end at any stage and we cannot find the answer.

For example,

$$0\overline{)\,8\,}\ \text{quotient } 1,\ \ 8-0=8 \qquad 0\overline{)\,8\,}\ \text{quotient } 2,\ \ 8-0=8 \qquad 0\overline{)\,8\,}\ \text{quotient } 3,\ \ 8-0=8$$

But no value work for 0 time because multiplication of any number with '0' is '0'. Therefore, the division of an integer by zero is not possible.

Remember

Zero (0) is an integer neither positive nor negative. When it is divided by any integer, the quotient is zero. But dividing any other integer is undefined.

Activity -3 Measure your body temperature for a week and write down your temperature of each day then show the result on number line also, find the difference between the highest and the lowest temperature.

3.5 VERIFICATION OF COMMUTATIVE, ASSOCIATIVE AND DISTRIBUTIVE LAWS

The following laws of multiplication of integers are true for

Commutative law of multiplication

According to this law of multiplication the product of two integers remains unchanged by changing their "order". For example

$$6 \times 5 = 5 \times 6$$
$$30 = 30$$

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Example 19 Verify the commutative law under multiplication for 24 and 28.

Solution By the commutative law of multiplication.

$$24 \times 28 = 28 \times 24$$

Take L.H.S	Take R.H.S
$= 24 \times 28$	$= 28 \times 24$
$= 672$	$= 672$

As, L.H.S = R.H.S So, $24 \times 28 = 28 \times 24$

Hence, commutative law of multiplication is verified.

Remember

Commutative law of division of integers is not true. e.g.

$$3 \div 2 \neq 2 \div 3$$

Associative law of multiplication

According to this law of multiplication "changing grouping" of the factors does not change the product. For example, $(4 \times 5) \times 7 = 4 \times (5 \times 7)$

$$20 \times 7 = 4 \times 35$$

$$140 = 140$$

Example 20 Verify the associative law for –6, 7 and 8.

Solution By the associative law of multiplication we need to verify

$(-6 \times 7) \times 8 = -6 \times (7 \times 8)$

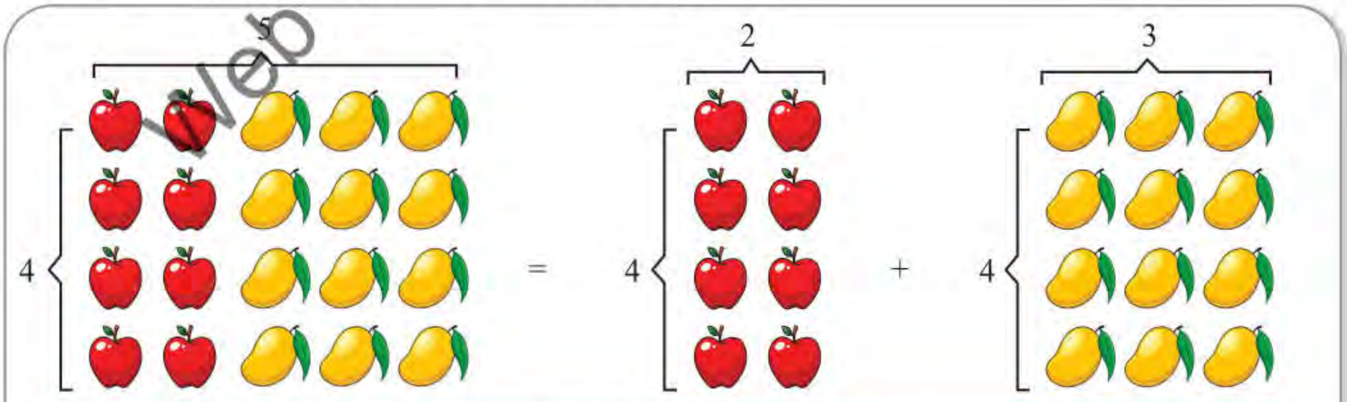
Take L.H.S	Take R.H.S
$= (-6 \times 7) \times 8$	$= -6 \times (7 \times 8)$
$= -42 \times 8$	$= -6 \times 56$
$= -336$	$= -336$

As, L.H.S = R.H.S

Hence, associative law of multiplication is verified.

Distributive law of multiplication

According to this law of multiplication, "multiplying a number by the group of numbers added together is same as doing each multiplication separately. For example,



$4 \times (2 + 3)$	=	4×2	+	4×3
4×5	=	8	+	12
20	=	20		

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Example 21 Verify the distributive law of multiplication for

$-10 \times (7 + 13) = (-10 \times 7) + (-10 \times 13)$

Solution

Take L.H.S	Take R.H.S
$= -10 \times (7 + 13)$	$= (-10 \times 7) + (-10 \times 13)$
$= -10 \times 20$	$= -70 - 130$
$= -200$	$= -200$

As, L.H.S = R.H.S

Hence, distributive law is verified.

Practice -8

(i) Verify commutative law for integers 105 and –301.

(ii) Verify the associative law of multiplication for integers –7, –13, –15.

(iii) Verify distributive law $-15 \times (8 - 7) = (-15 \times 8) + (-15 \times -7)$.

Did You Know?

The integer "1" has a special rule in multiplication just as the number "0" has in case of addition. "The product of "1" and an integer is the same as that number". For example,

$1 \times 13 = 13$ or $13 \times 1 = 13$

and the number 1 is known as multiplicative identity.

Exercise - 3.2

1. Find the product of following integers using number line:

(i) +4 and +5 **(ii)** –3 and +8 **(iii)** 0 and –6 **(iv)** +11 and –2

(v) +5 and –5 **(vi)** –10 and –3 **(vii)** +9 and 0 **(viii)** 2 and –7

2. Solve:

(i) $(+38) \times (-23)$ **(ii)** $(+24) \times (+15)$ **(iii)** $(-15) \times (-19)$

(iv) $(+23) \times (-41)$ **(v)** $(+74) \times (+0)$ **(vi)** $(-89) \times (-1)$

3. Find the quotient:

(i) $(+51) \div (+3)$ **(ii)** $(27) \div (-3)$ **(iii)** $(-54) \div (+9)$ **(iv)** $(-78) \div (-13)$

(v) $(+108) \div (-1)$ **(vi)** $(+156) \div 0$ **(vii)** $0 \div (+89)$ **(viii)** $(+1) \times (-228)$

4. Fill in the blanks with an appropriate integer:

(i) (_____) × (+7) = (–56) **(ii)** (–3) × (_____) = 30 **(iii)** (–21) ÷ (_____) = –3

(iv) (_____) ÷ (+12) = 60 **(v)** (+121) × (_____) = 0 **(vi)** (–212) ÷ (_____) = (–212)

5. Verify the following:

(i) $5 + 8 = 8 + 5$ **(ii)** $29 + 13 = 13 + 29$ **(iii)** $231 + 145 = 145 + 231$

6. Verify commutative law under addition for the following whole numbers:

(i) 29 and 39 **(ii)** 348 and 289 **(iii)** 828 and 215 **(iv)** 3335 and 411

7. Verify: **(i)** $-5 \times (-7 \times 8) = (-5 \times -7) \times 8$ **(ii)** $-9 \times (-10 \times -7) = (-9 \times -10) \times -7$

8. Verify the distributive law:

(i) $7 \times (8 + 9) = 7 \times 8 + 7 \times 9$ **(ii)** $10 \times (30 - 20) = 10 \times 30 - 10 \times 20$

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9. The sum of two integers is 194. One of them is –28. Find the other integer.

10. Rabia asked her mother to hold her savings. At the start of July her saving was Rs. 100. At the end of July she added Rs. 250 more in her savings. In the month of August she borrowed Rs. 185 to spend. What is the remaining or owed amount?

11. On Sunday morning the temperature of Faisalabad city was 45°C. It dropped 3°C per hour due to weather condition. What was the total change in temperature after 5 hours?

Project - 2 Create a model for distributive law of multiplication. You can use material from your surrounding e.g. straws, buttons, fruits and etc.

3.5 THE ORDER OF MATHEMATICAL OPERATIONS (BODMAS RULE)

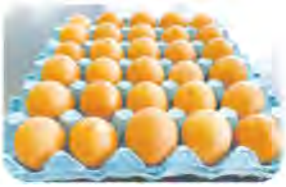
We are familiar with simplification of expressions from our previous classes. We have learnt about them. In routine life we add, multiply, subtract or divide two or more numbers for different purposes. After doing these we get our result in simplified form.

"The process of getting a simplified number is known as simplification".

To simplify the mathematical expression with two or more operations we use BODMAS rule. The letters of BODMAS are used for the order of operations which help us to simplify the given expressions. The order of operations used in the field of Mathematics, Science, Technology and many computer programming languages. For example,

Naseem has 30 eggs in each of six eggs trays. Seven other trays have 12 eggs in each tray. How many eggs does Naseem have in all?

To find how many eggs are there in all,



Naseem computes as follow $= \boxed{6 \times 30} + \boxed{7 \times 12}$

Total number of eggs $= 6 \times 30 + 7 \times 12$

$= 180 + 84$

$= 264$

Therefore, Naseem has 264 eggs in all.

Now, look at the given numerical expression. $5 \times (45 \div 7) - (18 \div 3) + 8$

To simplify the above type of mathematical expressions, we use BODMAS rule.

3.5.1 Different kinds of Brackets

Brackets are the symbols which are used in pairs to group two or more numbers together with mathematical operations.

There are four kinds of brackets that are mentioned below.

(i) Bar or vinculum. "——"

(ii) Parentheses or curved brackets or round brackets. "( )"

(iii) Braces or curly bracket. "{ }"

(iv) Square brackets or box brackets. "[ ]"

History

Achilles Reselfelt invented the BODMAS rule. To lean more about him, and the BODMAS rule visit the link.

https://www.africafeeds.com/2021/12/16/importance-of-bodmas-rule/

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3.5.2 Order of preferences as to remove the brackets from an expression

If any mathematical problem involving more than one bracket then brackets should be removed in the same orders. Look at the below example.

(i) Two students solve same question in different order.

(i) Student-I	**(ii) Student-II**
$30 \times (40 - \overline{24 + 10}) + 7$ $= 30 \times (40 - 24 - 10) + 7$ $= 30 \times (40 - 34) + 7$ $= 30 \times (6) + 7$ $= 180 + 7 = 187$ Right way	$30 \times (40 - \overline{24 + 10}) + 7$ $= 30 \times (40 - 24 - 10) + 7$ $= 30 \times (40 - 34) + 7$ $= 30 \times 40 - 34 + 7$ $= 1200 - 27 = 1173$ Wrong way

For preference of sign in an arithmetic expression, simply follow the below steps. Always proceed from left to right.

(i) $3 + 8 - 4$
$= 11 - 4 = 7$

(ii) $14 \div 2 \times 5$
$= 7 \times 5 = 35$

(ii) If there is a '+' sign before any pair of brackets, the brackets will be removed without changing the sign of the number within brackets. For example,

$3 + (6 - 2)$
$= 3 + (+4)$
$= 3 + 4 = 7$

(iii) If there is a '–' sign before any pair of brackets, the brackets will be removed by changing the sign of the number within brackets. For example,

$3 - (5 - 9)$
$= 3 - (-4)$
$= 3 + 4 = 7$

(iv) If there is no sign between a number (outside the pair of brackets) and a number within the brackets then there will be multiplication between them, and brackets will be removed after multiplication.

For example, $7(6 - 4)$
$= 7(2) = 7 \times 2 = 14$

The order of solving brackets within the given expression is mentioned below.

(i) "——" vinculum or bar is treated as first preference.

(ii) "( )" parentheses or curved brackets or round brackets are treated on second preference.

(iii) "{ }" curly brackets or braces are treated on third preference.

(iv) "[ ]" square brackets or box brackets are solved at the end.

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Example 22 Solve: $2020 - 18 [5 \times \{19 - 17 + (\overline{70 \div 10} + 5)\}]$

Solution

$= 2020 - 18 [5 \times \{19 - 17 + (7 + 5)\}]$	Vinculum solved
$= 2020 - 18 [5 \times \{19 - 17 + 12\}]$	Parentheses solved
$= 2020 - 18 [5 \times \{2 + 12\}]$	
$= 2020 - 18 [5 \times 14]$	Curly brackets are solved
$= 2020 - 18 [70]$	
$= 2020 - 1260$	Square brackets are solved
$= 760$	

3.5.3 Recognize BODMAS Rule

We have introductory discussion about the BODMAS rule in section 3.5.1. It shows the sequence, where more than one operations are performed in the simplification of mathematical expression.

In some circumstances we call it as, BIDMAS, BEDMAS, or PEDMAS created to remember of order of operations.

The letters of BODMAS, are used for the order of fundamental operations.

BODMAS		
B	For	"Brackets"
O	For	"of or order of operations"
D	For	"Division"
M	For	"Multiplication"
A	For	"Addition"
S	For	"Subtraction"

By using the BODMAS rule, we can simplify any given mathematical expression involving brackets and arithmetic operations. Now look at the following examples:

Example 23 Simplify: $3 \times 11 + 4 - 2 + (64 \div 8)$

Solution

$3 \times 11 + 4 - 2 + (64 \div 8)$	First compute expressions within the brackets
$= 3 \times 11 + 4 - 2 + (8)$	
$= 3 \times 11 + 4 - 2 + 8$	Remove brackets
$= 33 + 4 - 2 + 8$	According BODMAS, apply multiplication
$= 37 - 2 + 8$	Now, add or subtract from left to right
$= 35 + 8$	
$= 43$	

Example 24 Simplify: $4 \times 2 - [28 \div \{3 + (8 + \overline{6 - 3})\}]$

Solution

$4 \times 2 - [28 \div \{3 + (8 + \overline{6 - 3})\}]$	First compute bar or vinculum.
$= 4 \times 2 - [28 \div \{3 + (8 + 3)\}]$	Compute expression within round brackets.
$= 4 \times 2 - [28 \div \{3 + 11\}]$	Compute expression within braces.
$= 4 \times 2 - [28 \div 14]$	Compute expression within square brackets.
$= 4 \times 2 - 2$	Multiply or divide from left to right.
$= 8 - 2$	Add or subtract from left to right.
$= 6$	

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Example 25 Nabeel is a bookshop owner. He orders 30 new year diaries. The cost of each diary is Rs. 300. He paid Rs. 530 for shipping. How much Nabeel spent in all?

Solution

Total number of diaries	= 30
Cost of each diary	= Rs. 300
Shipping cost	= Rs. 530
Total spending	= (30 × 300) + 530
	= 9000 + 530 = 9530

Therefore, Nabeel spend Rs. 9530 in all.

Example 26 Waseem purchased the following items for his office. Four chairs at the rate of Rs. 950 each. Three tables at the rate of Rs. 2500 each. Two desktop computers at the rate of 4500 each. How much money did Waseem spend altogether?

Solution

Price of a chair	= Rs. 950
Total number of chairs	= 4
Price of four chairs	= 950 × 4 = Rs. 3800
Price of a computer table	= Rs. 2500
Total number of computer tables	= 3
Price of three computer tables	= 2500 × 3 = Rs. 7500
Price of a desktop computer	= Rs. 4500
Total number of desktop computers	= 2
Price of two desktop computers	= 4500 × 2 = Rs. 9000
Total amount paid by Waseem	= (950 × 4) + (2500 × 3) + (4500 × 2)
	= 3800 + 7500 + 9000 = 20,300

Remember

LCM stands for least common multiple.

Therefore, Waseem spent Rs. 20,300 altogether.

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3.5.4 Simplify the mathematical expressions involving fractions and decimals groupes

BODMAS rule can also use to simplify the mathematical expressions involving decimals and fractions. Now, look at the following examples:

Example 27 Simplify $14.75 + [3.20 - \{1.60 \div 2 \times (12.50 + 13.90 - 5.90)\}]$

Solution $14.75 + [3.20 - \{1.60 \div 2 \times (12.50 + 13.90 - 5.90)\}]$

$= 14.75 + [3.20 - \{1.60 \div 2 \times (12.50 + 8.00)\}]$

$= 14.75 + [3.20 - \{1.60 \div 2 \times 20.50\}]$

$= 14.75 + \left[3.20 - \left\{1.60 \times \frac{1}{2} \times 20.50\right\}\right]$

$= 14.75 + [3.20 - \{0.80 \times 20.50\}]$

$= 14.75 + [4.20 - 16.40]$

$= 14.75 - 12.20$

$= 2.55$

Example 28 One day Shabana went to Sunday bazar to buy vegetables with her mother. She bought 3 *kg* beetroot in Rs. 50 per *kg*, $\frac{1}{4}$ *kg* garlic in Rs. 200 per *kg*, $2\frac{1}{2}$ *kg* okra in Rs. 60 per *kg* and 5 *kg* onion in Rs. 20 per *kg*. How much money did she spend to buy all the vegetables?

Solution

Price of 1*kg* beetroot	= Rs. 50
Price of 3*kg* beetroot	= 3 × 50 = Rs. 150
Price of 1*kg* garlic	= Rs. 200
Price of 1*kg* garlic	$=\frac{1}{4}\times 200 =$ Rs. 50
Price of 1*kg* okra	= Rs. 60
Price of $2\frac{1}{2}$ *kg* okra	$=2\frac{1}{2}\times 60 = \frac{5}{2}\times 60 =$ Rs. 150
Price of 1*kg* onion	= Rs. 20
Price of 5*kg* of onion	= 5 × 20 = Rs. 100

By using the BODMAS rule.

Total price of all the vegetables = $(3\times 50)+\left(\frac{1}{4}\times 200\right)+\left(2\frac{1}{2}\times 60\right)+(5\times 20)$

$= 150 + 50 + 150 + 100 = 450$

Hence, Shabana spent Rs. 450 to buy all the vegetables.

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Activity -4 Visit the nearest pizza shop with your mother / father and ask the price of different types of pizza then calculate the total price of all types of pizza using mathematical operations.

(i) 3 large pizzas **(ii)** how many medium pizzas **(iii)** 6 small pizzas

Exercise - 3.3

1. Simplify the following:

(i) $4+6-7\times 15\div 3$ **(ii)** $25+(8\times 24\div 2-26)$

(iii) $44-\{26+(-84\times 84\div 42)\}$ **(iv)** $86\div(37+\{29-(114\div 6+9-5)\}]$

(v) $230\times\left[184-\left\{276\div 138\times 124\div\overline{4-30}\right\}\right]$ **(vi)** $221+\left[616-\left\{347+\left(435\div 3\times 2-\overline{36-6}\right)\right\}\right]$

2. Use the order of operations to simplify the following:

(i) $19.5+12.8-13.5$ **(ii)** $8.8\div 2.2-(-2.8)$

(iii) $30.8\div(3.1+0.4)$ **(iv)** $16.31+\{2.50-(5.94\div 2.20\times 4.0-1875)\}$

(v) $433.29-[505.25+\{113.18\div(89.5+24.97-1.29)\}]$

(vi) $5.50-\left[6.10\times 2-\left\{39+\left(4.5+\overline{8-2}\right)\right\}\right]$ **(vii)** $5.2+[0.5-\{6.4+(-9+5-3)\}]$

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3. Simplify the following by using order of operations:

(i) $3\frac{4}{99}+4\frac{7}{33}-5\frac{9}{11}$ **(ii)** $4\frac{8}{9}\times 3\frac{5}{14}\div 1\frac{1}{14}$ **(iii)** $\frac{7}{18}+\left(\frac{5}{9}\div\frac{1}{3}\right)$

(iv) $\frac{5}{9}+\left(\frac{7}{18}+\frac{1}{3}-\frac{1}{6}+\frac{1}{2}\right)$ **(v)** $\frac{8}{33}\times\left(\frac{9}{11}\div\frac{3}{22}\right)$ **(vi)** $\frac{5}{12}+\left\{\frac{11}{24}-\left(\frac{2}{3}+\overline{\frac{7}{8}-\frac{2}{3}}\right)\right\}$

(vii) $-3\frac{9}{24}\div\left[4\frac{3}{8}\times\left\{\frac{1}{2}+\left(5\frac{1}{2}-6\frac{1}{2}\right)\right\}\right]$ **(viii)** $1\frac{5}{7}$ of $21+3\frac{3}{4}\div\frac{3}{7}-1\frac{1}{3}$

4. There are 320 students in a school. 220 students receive pocket money Rs. 50 per day and the rest receive Rs. 70 per day. To the management of school, a week is equal to six working days. How much amount all the students spend in a week?

5. A factory worker makes 90 bars of soap in a week and sells these at the rate of Rs. 250 each. Before the soap can all be sold, the workers found out 10 bars were nibbled by mice. How much will be the total sale at the end of a four week month?

6. Shazmeena had 30.50 metres of cloth. She used 12.60 metres of cloth for her suit. She gives 10.80 metres of cloth to her friend. How much cloth is left with Shazmeena?

7. The price of 20 *kg* chemical is Rs. 25000. Find the price of 13 *kg* chemical.

8. Nasreen bought 15 metres of cloth from the market. She used half of the cloth for her suit, $\frac{1}{3}$ part of it for her little daughter. She used remaining cloth for her nieces's suit. How much cloth did she use for her nieces's suit?

9. Sakeena required $24\frac{1}{4}$ metres long wire for a connection. She joined three length of wires $4\frac{3}{4}$ metres, $5\frac{1}{8}$ metres and $6\frac{1}{2}$ metres. How much more length of wire she needed?

Project - 2 Create your own word problem in which four operations are used.

Review Exercise 3

1. Each of the following question is followed by four suggested options. In each case select the correct option.

(i) The inverse process of addition is called:

a. addition b. subtraction c. multiplication d. division

(ii) The product of odd number of integers is always:

a. odd b. odd and even c. even d. prime

(iii) The product of three negative integers is always:

a. positive b. negative c. even d. odd

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(iv) The sign of the product $(-6)\times(-6)\times(-6)\times(-6)\times(-6)\times(-6)\times(-6)$ is:

a. – b. + c. × d. ÷

(v) $-150\times -3 =$ ________________

a. –50 b. +450 c. –450 d. –100

(vi) The process of getting a simplified number is called:

a. purification b. simplification c. explanation d. exploration

(vii) [ ] brackets are known as:

a. vinculum b. braces c. perantheses d. box brackets

(viii) If all the brackets are found in a given expression, then the first bracket which will be removed is:

a. { } b. ( ) c. ——— d. [ ]

(ix) If a word 'of' exists in any expression it means:

a. addition b. subtraction c. multiplication d. division

(x) While solving a mathematical expression always use BODMAS rule and proceed from:

a. right to left b. left to right c. middle to left d. left to middle

(xi) In the given mathematical expression if all the brackets are involved, we will solve ________ bracket at the end:

a. ——— b. ( ) c. { } d. [ ]

(xii) According to the BODMAS rule, first basic operation is performed:

a. addition b. subtraction c. multiplication d. division

(xiii) $(33.5 + 90 - 70)\times 0$:

a. – 254.5 b. 254.5 c. 0 d. 147

(xiv) $\frac{3}{4}+\left(\frac{5}{7}-\frac{2}{7}\div\frac{2}{7}\right)$

a. $1\frac{1}{22}$ b. $1\frac{1}{21}$ c. $\frac{1}{28}$ d. $\frac{13}{28}$

2. Give the short answer for each question:

(i) What are like integers?

(ii) What are unlike integers?

(iii) Write commutative law with respect to addition.

(iv) Write associative law with respect to addition.

(v) What is associative law of multiplication?

(vi) What is distributive law of multiplication?

(vii) What is BODMAS rule?

3. Simplify the following:

(i) $80 - [25 - \{20 - (7 - \overline{9 - 3})\}]$

(ii) $100 - [289 \div 17 \times 7) - (-5) - \{2 - \overline{17 - 10})\}]$

(iii) $3 \div 15 + [2.4 - \{0.50 \times (8.50 - \overline{4.60 - 2.60}\}]$

(iv) $4\frac{3}{11}+\left[5\frac{5}{8}\div\left\{\frac{3}{5}+\left(4\frac{1}{5}\times\overline{2\frac{1}{6}-3\frac{1}{6}}\right)\right\}\right]$

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4. The sum of two integers is –215. If one of them is –135, then find the other.

5. The product of two integers is –180. If one of them is 10, then find the other.

6. Find an integer that divides –150 to give –3.

7. The difference of two integers is 185. If one of them is 400, then find the other integer.

8. The sum of two integers is –105. If one of them is 95, then find the other.

9. Anee is reading a book from the past three weeks. She reads 35 pages everyday. She still has 200 pages left. How many pages does the book have in all?

10. In how much time Raheel will cover 60 kilometres distance with a speed of 10 kilometre per hour?

11. Akbar is a shopkeeper. He sold $8\frac{1}{4}$ *kg* rice, $6\frac{1}{4}$ *kg* sugar and $12\frac{1}{2}$ *kg* flour. How much did he sell in all?

12. Neelam has Rs. 1500 balance in her mobile. She gave $\frac{3}{5}$ to Naila. Naila gave $\frac{1}{5}$ of her share to Nimra. How much share Nimra has?

Project - 3

Calculate the age of your all siblings in days then find the difference between the youngest and the oldest ages by using BODMAS rule.

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Summary

- The integers with the same signs are called like integers.
- The integers with different signs are called unlike integers.
- According to the commutative law of addition, the sum of two integers remains unchanged by changing the "order"
- Zero (0) is an integer neither positive nor-negative.
- According to the associative law of addition changing the "grouping" of addends does not change the sum.
- According to the commutative law of multiplication the product of two integers remains unchanged by changing their "order".
- According to the associative law of multiplication "changing grouping" of the factors does not change the product.
- According to the distributive law of multiplication multiplying an integer by the group of integers added together is same as doing each multiplication separately.
- The product of two unlike integers is always negative.
- The product of two like integers with negative signs is always positive.
- Division of integer by zero (0) is not possible.

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- The word BODMAS indicates the sequence in which more than one operations are performed in the simplification of mathematical expressions.
- If there is a plus '+' sign before any pair of the brackets, the brackets will be removed without changing the sign of the numbers within brackets.
- If there is a plus '–' sign before any pair of the brackets, the brackets will be removed with changing the sign of the number within brackets.
- If there is no sign between a number (outside the pair of brackets) and a number within the brackets, then there will be multiplication between both numbers, and brackets will be removed after multiplication.
- Brackets are solved in the following order 1st ——— vinculum of bar, 2nd ( ) parentheses, 3rd { } braces and 4th [ ] square brackets.

Challenge

Look at the magic squares and complete each square such that the numbers in each square add up to the same sum horizontally, vertically or diagonally.

		–4
	–1	1
2		

		2
5	1	
0		

–5		
–7	–5	–3
		–4

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Teaching Point

The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

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Sub-Domain 4

RATE, RATIO AND PERCENTAGE

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Define and calculate ratio, equivalent ratio, rate and percentage.
- Solve real life problems involving ratio, rate and percentage.

How many eggs can be placed in four plates? If there are

12 eggs in two plates 18 eggs in three plates

4.1 RATIO

"The ratio is a numerical comparison between two quantities or things of same kind."

In our daily life we commonly use the word ratio while distributing the quantities, money and other things. for example,

There are four green mangoes to five yellow mangoes.

A ratio can be expressed in three forms, **(i)** Word form **(ii)** Ratio form **(iii)** Fraction form

The above example of mangoes shows.

4 to 5 in words form

4 : 5 in ratio form

$\frac{4}{5}$ in fraction form.

Remember

Always compare the quantities of same kind with same unit. Beacuse the ratio of two same quantities can be defined, but the ratio of two different quantities is meaningless. e.g. We can not compare his/her weight with his/her height.

Each form is read as *"the ratio of 4 to 5"* similarly in general we use 'x' for the first quantity and 'y' for the second quantity and write them as $x : y$ as *"the ratio of x to y".*

If more than two quantities are given we write them as $x : y : z$ in the form of ratio.

4.1.1 Defining a Ratio

Every country of the world has its national flag. A national flag is a flag that represents and symbolizes a country. Look at the Pakistan's National flag. It is in the ratio of height to length. Which is 2 : 3.

So for every 2 (centimetre, metre or feet) whatever of height there must be 3 of length.

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Example 1 If we make a flag of 20 *cm* high, it must be 30 *cm* long.
If we make another flag of 40 *cm* high it must be 60 *cm* long.

But both times flag is still in the ratio 2 : 3 so, we can say

"The ratio is a relation in which one quantity bears to another quantity of the same kind with regards to their magnitude".

4.1.2 Antecedent and Consequent in a Ratio

In previous section 4.1 we compared the green mangoes with yellow mangoes. The ratio of green to yellow is 4 : 5. Here in this example the number 4 is the first element in the ratio and the number 5 is the second element in the ratio.

"The first element is called antecedent and the second element is called consequent."

Antecedent → 4 : 5 ← Consequent

Remember
- Order of ratios always matters, i.e 4:5 is not same as 5:4.
- Ratio is unit less or it has no unit.

Example 2 Write down the antecedent and consequent of the following ratios:

(i) 7 : 9 (ii) a : b (iii) 19 : 20

Solution

(i) 7 : 9
Antecedent = 7
Consequent = 9

(ii) a : b
Antecedent = a
Consequent = b

(iii) 19 : 20
Antecedent = 19
Consequent = 20

4.1.3 Calculation of Ratio

Ratios are the mathematical expressions in which we compare two or more quantities. Ratios can be calculated and written in several different ways. But principle ways have been discussed in the following examples:

Example 3 Nimra wants to celebrate her birthday. She used 8 table spoons milk and 24 table spoons unsalted butter to make her birthday cake. Write the ratio of milk to unsalted butter.

Solution Milk used for birthday cake = 8 table spoons
Unsalted butter used for birthday cake = 24 table spoons.
So, the ratio of milk to unsalted batter is 8:24 = 1:3

Example 4 There are 400 students and 40 teachers in a school. Write the ratio of teachers to students.

Solution Number of teachers in the school = 40
Number of students in the school = 400
So, The ratio of teachers to students is 40 : 400 = 1 : 10

Activity -1

Make a national flag with the ratio 2:3. Your flag measurement should be different from your friend's flag measurement.

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4.1.4 Simplification of Given Ratio into Lowest (Equivalent) Form

We know that a ratio is another form of common fraction. To reduce the given ratios into the lowest (equivalent) form we divide each term of ratio by the H.C.F (Highest Common Factor) of all the terms. Look at the following examples:

Example 5 Reduce the following ratios into the lowest equivalent ratios.

(i) 7 : 56 (ii) 9 : 36 : 48

Solution

(i) 7 : 56

H.C.F of 7 and 56 is 7

Now, divide each term of the ratio by 7.

$$= \frac{7}{7} : \frac{56}{7}$$
$$= 1 : 8$$

Hence, the required lowest (equivalent) ratio is 1: 8.

(ii) 9 : 36 : 48

HCF of 9, 36 and 48 is 3

Now, divide each term by 3.

$$= \frac{9}{3} : \frac{36}{3} : \frac{48}{3}$$
$$= 3 : 12 : 16$$

Hence, the required ratio is 3: 12 :16.

Thinking Time If Muddasar is 10 years old and Humaira is 8 years old. What is the ratio of their ages? What is the ratio of their ages after five years?

Remember A ratio remains unchanged when both the terms are multiplied or divided by the same number. This is known as equivalent ratio.

Example 6 There are 40 boys and 60 girls in two sections of Mathematics class. Find the ratio of boys to girls in both sections, also write the answer in the simplest form.

Solution

Number of boys in Mathematics class = 40

Number of girls in Mathematics class = 60

boys : girls

40 : 60

H.C.F of 40 and 60 is 20

Now, divide each term of ratio by 20.

$$= \frac{40}{20} : \frac{60}{20}$$
$$= 2 : 3$$

Hence, the ratio of boys to girls is 2:3.

Practice -1

1. Simplify the following ratios: (i) 60 : 2000 (ii) 25 : 625
2. Rehman earns Rs. 500 in an hour and spend Rs. 350. Find the ratio of his income to expenditure.

4.1.5 Relationship Between Ratio and Fraction

We have discussed in section 5.1 that the ratio can also be expressed in fraction form. Where the numerator is called antecedent and the denominator is called consequent.

$$3 : 5 = \frac{3}{5}$$

Similarly, in general $a : b = \frac{a}{b}$ where 'a' is antecedent and 'b' is consequent.

A ratio has no unit. It is just a number which indicates how many times one quantity is greater than the other.

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Example 7 First write in fraction form and then simplify.

(i) 18 : 24 **(ii)** 40 : 70 **(iii)** 25 : 45

Solution

(i) 18 : 24

$= \frac{18}{24}$

$= \frac{3}{4}$

Hence, 18 : 24 = 3 : 4

(ii) 40 : 70

$= \frac{40}{70}$

$= \frac{4}{7}$

Hence, 40 : 70 = 4 : 7

(iii) 25 : 45

$= \frac{25}{45}$

$= \frac{5}{9}$

Hence, 25 : 45 = 5 : 9

Thinking Time

Suppose you grew at a rate of one feet per year. About how many feet tall would you be today?

Practice -2 Write the given ratios in the form of fraction and simplify: **(i)** 7 : 11 **(ii)** 14 : 29

Example 8 Find the ratio of the following and write in the simplest form:

(i) $\frac{5}{6}$ and $\frac{2}{9}$ **(ii)** $1\frac{5}{6}$ *km*, $1\frac{11}{15}$ *km*, and $1\frac{13}{30}$

Solution

(i) $\frac{5}{6}$ and $\frac{2}{9}$

$\frac{5}{6} : \frac{2}{9}$

$= \frac{5}{6} \times 18 : \frac{2}{9} \times 18$ (multiply both by LCM of 6 and 9)

$= 5 \times 3 \; : \; 2 \times 2$

$= 15 : 4$

(ii) $1\frac{5}{6}$ *km*, $1\frac{11}{15}$ *km*, and $1\frac{13}{30}$

$1\frac{5}{6}\,km : 1\frac{11}{15}\,km : 1\frac{13}{30} = \frac{11}{5}\,km : \frac{26}{15}\,km : \frac{43}{30}\,km$

$= \frac{11}{5} \times 30 : \frac{26}{15} \times 30 : \frac{43}{30} \times 30$ (multiply both by LCM of 5, 15 and 30)

$= 66 : 52 : 43$

Example 9 Naseem has Rs. 1600. She shared this amount with her two friends Rabia and Fatima in the ratio 3 : 5. How much amount each of them received from Naseem?

Solution Total amount with Naseem = Rs. 1600

The sum of money divided into $3 + 5 = 8$ equal parts.

Rabia and Fatima received 3 and 5 parts respectively.

Rabia received $= \frac{3}{8}$ of $1600 = \frac{3}{8} \times 1600$

$= 3 \times 200 =$ Rs. 600

So, Rabia received Rs. 600 from Naseem.

Similarly, Fatima received $= \frac{5}{8}$ of $1600 = \frac{5}{8} \times 1600$

$= 5 \times 200 =$ Rs. 1000

So, Fatima received Rs. 1000 from Naseem.

Practice -3 Find the ratio of :

(i) 500 g to 1 kg **(ii)** $\frac{3}{5}$ to $\frac{7}{9}$

Check

Rabia + Fatima = 1600

600 + 1000 = 1600

1600 = 1600

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4.1.6 Continued Ratio

"As, the ratio is a numerical comparison between two quantities or things of same kinds". If 'a' and 'b' are two quantities having same units where $b \neq 0$ then the ratio of a and b can be written as,

$a : b$ (read as "a ratio b")

Sometime we compare more than two quantities.

"The comparison among three or more quantities is called continued ratio."

If a, b, c, d, e and f are in continued ratio then $a : b = b : c = c : d = d : e = e : f$

In fraction form $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f}$

There are many situations in our every day life where we use continued ratio such as,

(i) Ratio among the shares of profit between three or more persons.

(ii) Ratio among the prices of three or more different types of cars etc.

Remember

Always compare the same quantities with same units.

Example 10 Nadia wants to distribute Rs.9435 among her three children. Nida, Rehan and Aleem. She distributed this money in such a way that Nida : Rehan = 4 : 5 and Rehan : Aleem = 3 : 2. How much money each child will get?

Solution

Total amount with Nadia = Rs. 9435

The ratio of Nida, Rehan and Aleem is,

Nida	:	Rehan	:	Aleem
4	:	5		
		3	:	2
4 × 3	:	3 × 5	:	5 × 2
12	:	15	:	10

Sum of ratios = 12 + 15 + 10 = 37

Nida's share $= \frac{12}{37} \times 9435$

= Rs. 3060

Rehan's share $= \frac{15}{37} \times 9435$

= Rs. 3825

Aleem's share $= \frac{10}{37} \times 9435$

= Rs. 2550

Check

To check our calculation we will add all shares.e.g 3060 + 3825 + 2550 = Rs. 9435

Practice -1 The total price of three electronic items is in the ratio washing machine to fridge is 2 : 3 and fridge to AC is 4 : 5. If Saba paid Rs. 98000, then find the price of each item.

Remember To solve any type of word problem follow the below four steps.

(i) Read: Read the statement of given problem and involve yourself in it until you understand.

(ii) Plan: After understanding plan to find the required things and decide which method you need to apply for your solution.

(iii) Solve: Make a calculation and find the required thing from the given facts.

(iv) Check: Finally check your solution by applying different methods or revise your solution.

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Activity -1 Ask the value of the given prize bond from your parents and then divide the valued amount among your all siblings in the ratio boys to girls as 1 : 2

Exercise - 4.1

1. The table shows the obtained marks of grade – 6 students in Mathematics test. Write down the each given ratio of obtained marks in other two forms.

Name of pupil	Marks
Shoukat	55
Saima	62
Waseem	46
Ali	85
Rabia	70
Hiba	35
Arslan	50
Zeeshan	80

(i) Zeeshan to Arslan.
(ii) Hiba to Shoukat.
(iii) Waseem to Rabia.
(iv) Ali to Arslan.
(v) Shoukat to Ali.
(vi) Shoukat to Ali to Arslan.
(vii) Waseem to Saima to Rabia.

2. Write each of the following into ratio form:

(i) $\frac{3}{5}$ **(ii)** $\frac{7}{12}$ **(iii)** $\frac{34}{37}$ **(iv)** $\frac{53}{59}$

(v) $\frac{101}{103}$ **(vi)** $\frac{x}{y}$ **(vii)** $\frac{x}{x+1}$ **(viii)** $\frac{a-b}{p+q}$

3. Express each ratio into the fraction form and then simplify:

(i) 50 : 60 **(ii)** 55 : 65 **(iii)** 30 : 60 **(iv)** 85 : 90

(v) 75 : 85 **(vi)** 85 : 95 **(vii)** 56 : 70 **(viii)** 0.6 : 0.9

4. During a PSL (Pakistan Super League) match Naveed completed his century in one hour while Sarfraz took 1 hour 30 minutes. What is the ratio of time taken by both the players to reach their centuries. Write the ratio in the simplest form.

5. Aneel's monthly salary is Rs. 55,000. He spends his monthly salary in the ratio 2 : 3 as saving and expenditure respectively. Find his monthly expenditure and saving.

6. Distribute Rs. 99,000 among three brothers Moeez, Zurain and Abdul Hadi. Such that Moeez : Zurain = 4 : 5 and Zurain : Abdul Hadi = 5 : 6.

Project - 1 Write your one month total pocket money and make a calculation for each of your sibling's share in the ratio 1 : 2.

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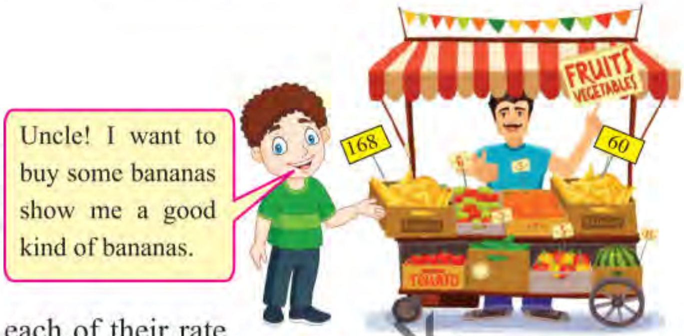
4.2 RATE



Let's help Ali to buy bananas from the fruit shop.

The first kind of bananas cost is Rs. 168 for 12 bananas. The second kind of bananas cost is Rs. 60 for 6 bananas. Which kind of bananas Ali should buy?

To find the result of this problem, we have to find each of their rate for an equal number of bananas, such as one banana.

Price of the first kind of a banana $= \frac{168}{12} =$ Rs. 14

Price of the second kind of a banana $= \frac{60}{6} =$ Rs. 10

Thus, we should suggest Ali to buy the second kind of bananas.

From the above example we concluded that

"A rate is a special kind of ratio in which the two terms are in different units."

Remember

A ratio is comparison of two or more numbers or things with same units. But the rate is special kind of ratio in which we compare two or more quantities with different units.

Example 11 Shazmeena is buying jellybeans for her sister Sofia's birthday party. She buys a bag of 50 g of jellybeans that cost is Rs. 500. Shazmeena is wondering about the cost of 1 g of jellybeans. How can she find the cost of 1 g of jellybeans?

Solution Cost of 50 g of jellybeans = Rs. 500

Cost of 1 g of jellybeans = ?

To find the cost of 1 g of jellybeans we need to divide as: $\frac{500 \div 50}{50 \div 50} = 10$

Hence, the cost of 1g of jellybeans is Rs. 10

Practice -2 Nadeem pays Rs. 50 for parking his motorcycle in a shopping mall for 5 hours. Calculate the parking charges for 3 hours and 30 minutes.

Exercise - 4.2

1. Nimra works in an office as a secretary. Last week she worked 35 hours and earned Rs.29,995. Calculate.

(i) How much amount Nimra earns per hour? (ii) How much amount Nimra earns for 10 hours?

(iii) How much amount Nimra earns for 25 hours?

2. Naureen walked 2 km and 500 m in 50 minutes. What was her rate of walking in kilometre per hour?

3. Rehan drove his car at an average rate of 80 km per hour. How far did he drive in 5 hours?
   Hint: Use formula; Distance = rate × time

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4. Shumail takes $2\frac{1}{2}$ hours to paint a 15 *m* long wall. What is his rate of painting wall per hour?

5. 7 *kg* of cooking oil cost is Rs. 2415. What is the cost of 5 *kg* of cooking oil?

6. Naeem has to pay Rs. 875 for 350 minutes of outgoing calls by using his mobile. Find:

(i) How much amount is charged for 7 minutes of outgoing calls?

(ii) The amount he has to pay if he makes 563 minutes of outgoing calls.

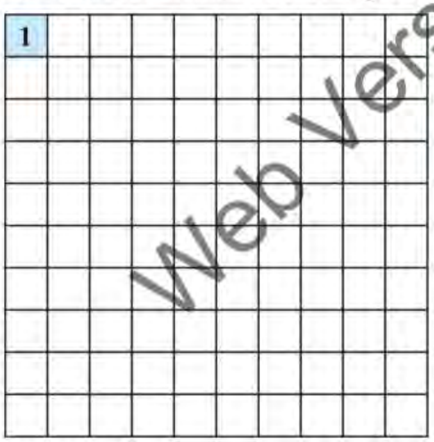
Activity -2 Visit the nearest shopping mall/store with your parents. Get the price of 10 *kg* Banaspati Ghee teen pack. Then calculate how much total amount you need to pay for 12 *kg* of Banaspati Ghee. Show your calculations to your teacher.

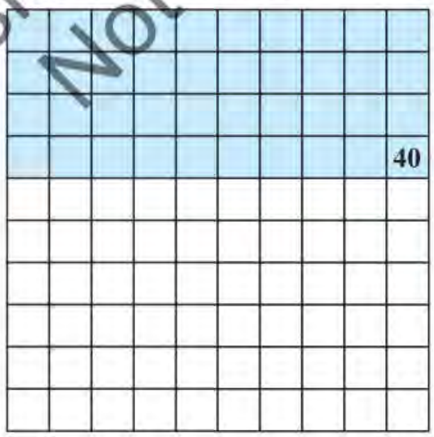
4.3 PERCENTAGE

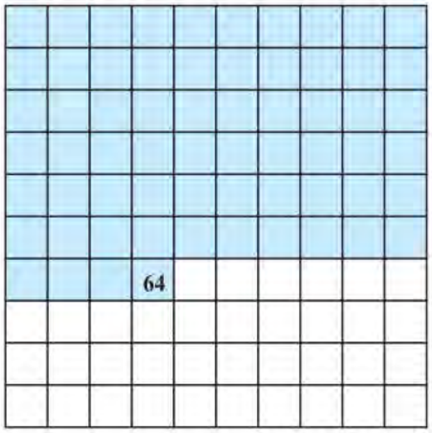
We are familiar with the word percentage. We have learnt about it in our previous grade and normally heard the phrases like sale, sale up to 70% off in our daily life. We apply percentages on many things, like percentage discount, percentage of your obtained marks, etc. percentage is a short form of Latin word *'per centum'* meaning out of hundred which means the ratio of some numbers to hundred. A symbol % is used for percent.

4.3.1 Percentage as a Fraction

Percent means the ratio of a given number to hundred. Look at the following figures:







1 out of 100 of the squares is coloured. So, "1 per 100" or $\frac{1}{100}$ means 1%.

40 out of 100 of the squares coloured. So, "40 per 100" or $\frac{40}{100}$ means 40%.

64 out of 100 of the squares coloured. So, "64 per 100" or $\frac{64}{100}$ means 64%.

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4.3.2 Expressing One Quantity as Percentage of Another

It is very common to write one number as percentage of another number. For example, it's used for calculating percentage of your obtained marks in a test or exam or working out costs on a project etc.

> **Remember**
>
> $100\% = \frac{100}{100}$. It means multiplying by 1 does not change the value.

Generally, to express one quantity as a percentage of another, write the given quantity as a fraction of the total and multiply it by 100% or $\frac{100}{100}$. Both the quantities expressed in the same unit.

Example 12 Waqar obtained 180 marks in his 1^{st} term exams. The total marks of his all subjects are 200. Find the percentage of Waqar's obtained marks.

Solution

Total marks of all subjects = 200

Obtained marks = 180

$$\textit{Percentage of obtained marks} = \frac{\textit{obtained marks}}{\textit{total marks}} \times 100 = \frac{180}{200} \times 100$$

$$= \frac{180}{200} \times 100 = 90\%$$

Hence, Waqar obtained 90% marks in his 1^{st} term examination.

> **Practice -6**
>
> In a school, there are 13 male teachers and 20 female teachers. Calculate the percentage of
> (i) male teachers
> (ii) female teachers

> **Challenge**
>
> There are 97 boys and 3 girls reached in a school on time. If the total strength of school is 215 boys and 135 girls. What is the percentage of boys and girls that did not reach on time?

4.3.3 Percentage of Given Quantities

The method of finding the percentage has been illustrated in the following examples:

Example 13 Find the percentage of the following numbers:

(i) 6% of 100 (ii) 25% of 320 (iii) 42% of 1000

Solution

(i) 6% of 100

$= \frac{6}{100}$ of 100

$= \frac{6}{100} \times 100$

$= 6$

(ii) 25% of 320

$= \frac{25}{100}$ of 320

$= \frac{25}{100} \times 320$

$= 80$

(iii) 42% of 1000

$= \frac{42}{100}$ of 1000

$= \frac{42}{100} \times 1000$

$= 420$

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Example 14 Qadeer has Rs. 2000. Which is 40% of a new cycle. How much more money does he have to arrange for the cycle?

Solution 40% of total price of cycle = 2000

Total quantity = Given quantity ÷ percentage.

$$\text{Total price of cycle} = 2000 \div 40\%$$

$$= 2000 \div \frac{40}{100}$$

$$= 2000 \times \frac{100}{40}$$

$$= 5000$$

So, the total price of cycle is Rs. 5000

More required money = 5000 – 2000 = 3000

Hence, Qadeer needs Rs. 3000 more to buy a new cycle.

> **Remember**
> To calculate the total quantity always use the formula.
> Total quantity = Given quantity ÷ percentage.

Activity -3 Calculate the percentage of your previous grade obtained marks.

Practice -3 Find the value of each to the right. **(i)** 15% of 150 **(ii)** 1000% of Rs. 13.50

4.3.4 Comparison of two Quantities by Percentage

As, both the students from different schools are sharing the final exams result of their schools. Since 550 > 380, does this mean that Alina's school result is better than Ali's school result? To handle this type of situations, we find the percentage result of both schools and compare their results.e.g

Alina's school result	Ali's school result
Percentage result = $\frac{550}{600} \times 100\%$ = 91.67%	Percentage result = $\frac{380}{400} \times 100\%$ = 95%

As, the percentage result of Ali's school is greater than the Alina's school result.
Therefore, Ali's school performed better.

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Example 15 There are two bags, A and B. Bag A contains 80 white marbles out of 350 total marbles. Bag B contains 65 white marbles out of 200 total marbles. Which bag contains higher percentage of white marbles?

Solution

Bag-A

No. of white marbles = 80

Total no. of marbles = 350

White marbles percentage = $\frac{80}{350} \times 100\%$

$= 22.86\%$

Bag-B

No. of white marbles = 65

Total no. of marbles = 200

White marbles percentage = $\frac{65}{200} \times 100\%$

$= 32.5\%$

From the above calculation we can conclude that bag B contains higher percentage of white marbles.

Practice -4 In an election, candidate A got 14570 votes out of 20,000 total votes. Candidate B got 13495 votes out of 15000 total votes. Which candidate got more votes?

(i) Candidate A (ii) Candidate B Also, calculate the percentage votes of each candidate.

4.3.5 Increasing or Decreasing of a Quantity by a Given Percentage

In our routine life, we observe changes happen everyday e.g, in the month of February, one day the temperature outside the home was warm and pleasent. Aslam wore a T-shirt and went to play cricket outside his house. But then on the next day it was cold and Aslam ends up wearing his sweater with pants. Similarly, changes take place in business and other field of life.

We calculate percentage increase/decrease and express the increase/decrease amount as percentage.

The procedure to calculate the increasing or decreasing of a quantity by a given percentage is illustrated in the following examples:

Example 16 A shopkeeper increases the price of an article by 10% from its original price. If the original price of the article is Rs. 5650, what will be the new price of the article?

Solution

An increase of 10% means for every Rs. 100 of the original price, there is an increase of Rs. 10. So, each Rs. 100 in the original price becomes Rs. 110 in the new price.

Therefore, new price : original price = 110 : 100

The fraction form is $\frac{\text{New price}}{\text{Original price}} = \frac{110}{100}$

New price $= \frac{110}{100} \times$ original price

$= \frac{110}{100} \times 5650 =$ Rs. 6215

Increase in price = new price – original price

= Rs. 6215 – Rs. 5650

= Rs. 565

Hence, the shopkeeper has increased Rs. 565 in the original price.

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Example 17 The original price of petrol is Rs. 161 per litre. If the government decreases its price by 12% then what will be the new price. Also, calculate the decreased amount.

Solution

A decrease of 12% means for every Rs. 100 of the original price, there is a decrease of Rs. 12. e.g each Rs. 100 in the original price becomes Rs. 88 in the new price.

So, new price : original price = 88 : 100

The fraction form is $\dfrac{\text{New price}}{\text{Original price}} = \dfrac{88}{100}$

New price = $\dfrac{88}{100} \times$ original price

$= \dfrac{88}{100} \times 161 = \text{Rs. } 141.68$

Decrease in price = original price – new price

$= 161 - 141.64$

$= \text{Rs. } 19.32$

Hence, the decreased amount is Rs. 19.32

Remember In general we use the formulas

(i) New value = final percentage × original value (ii) Increase/decrease = percentage increase/decrease × original value

Example 18 The price of calculator is decreased from Rs. 2000 to Rs. 1340 but the price of school bag is raised from Rs. 1799 to Rs. 2295. Find the percentage increase and percentage decrease of both prices.

Solution

Calculator

Original price = Rs. 2000

New price = Rs. 1340

Decreased amount = 2000 – 1340

= Rs. 660

The percentage decrease

$= \dfrac{\text{Decrease}}{\text{Original price}} \times 100\%$

$= \dfrac{660}{2000} \times 100\%$

$= 33\%$

Hence, the price of calculator is decreased by 33%.

School Bag

Original price = Rs. 1799

New price = Rs. 2295

Decreased amount = 2295 – 1799

= Rs. 496

The percentage decrease

$= \dfrac{\text{Increase}}{\text{Original price}} \times 100\%$

$= \dfrac{496}{1799} \times 100\%$

$= 27.6\%$

Hence, the price of school bag is increased by 27.6%.

Practice -5

(i) Increase 65 by 130%, what is the result?

(ii) Decrease 125 by 65%, what is the result?

(iii) In 2019, Naeem was given 10% increment in his basic salary of Rs. 25500. Find his new salary after increment.

(iv) In 2020, due to pandemic of Covid-19 Nadia suffered a pay cut off 20% in her basic pay of Rs. 30,000. Find her new salary after cut off.

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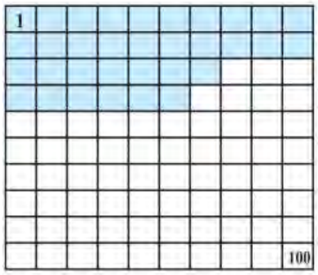
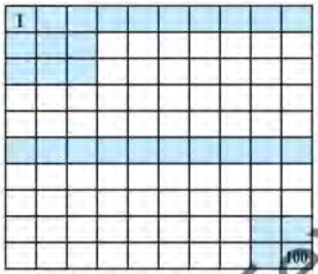
Activity -4 Find percentage change in your grade-4 and grade-5 marks.

Exercise - 4.3

1. Write the percentage of the following coloured part of each figure:

(i) 

(ii) 

(iii)

2. Write the following percentages into the simplified form of fraction:

(i) 25% (ii) 20% (iii) 84% (iv) 34% (v) 93%

3. Solve the following:

(i) 8% of 100 (ii) 15% of 60 litres (iii) 35% of 160 (iv) 80% of 900

(v) $9\frac{1}{4}\%$ of 1000 (vi) $3\frac{3}{8}\%$ of 80 (vii) 9.8% of 80 (viii) 8.9 % of 900

4. Nimra obtained 125 marks out of 250. Calculate the percentage of obtained marks of Nimra.

5. There are 400 students in a school. 30% of them are left handed. How many students are right handed?

6. 65% of the students come from Govt. High School Model Town. If there are 1500 studetns in the school, then find how many students are not from Model Town?

7. Your Mathematics class is of 40 minutes duration. If your teacher spends 15 minutes to check copy work and remaining time to complete a mathematical activity. Find the percentage of:

(i) Time spent for copy work. (ii) Time spent for mathematical activity.

8. Anee spends 80% of the total earning at the golf course last summer. If she spends Rs. 35,000, then find her total earning at golf course.

9. The 20% of a number is 120. Find the number.

10. The 70% of a tree is 3.8 *m*. Find the total height of the tree.

11. 20% of the students of a school were absent on a certain day. If there were 1200 students present in the school on that day, then find the total number of students.

12. Nadia used 6 *kg* of Banaspati Ghee out of 8 *kg* Banaspati Ghee in the month of March. She used 7 *kg* of Banaspati Ghee out of 10 *kg* Banaspati Ghee the month of April. In which month she used more Ghee than the other month?

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13. In the month of Ramadan, Saleem donated Rs. 4500 to charity from his total salary of Rs. 45000. Aleem donated Rs. 6000 to charity from his salary of Rs. 55000. Who has donated the higher percentage of his monthly salary?

14. Increase 17 by 85%, what is the result?

15. Decrease 30 by 70%, what is the result?

16. In the month of September Salma got 15% increase in her monthly salary. If Salma's monthly salary is Rs. 45000, then what is her new salary?

17. After 12% of a bill has been deducted, Rs. 6536 remains to be paid. How much was the original bill?

18. The cost of a bicycle is raised from Rs. 8500 to Rs. 10,000. What is the percentage increase in the price of bicycle?

Project - 3

Find the percentage increase/decrease in your previous two months electricity bills.

Go Online

Visit the below link to play games and practice percentage increase or decrease.
https://www.quia.com/rr/230204.html

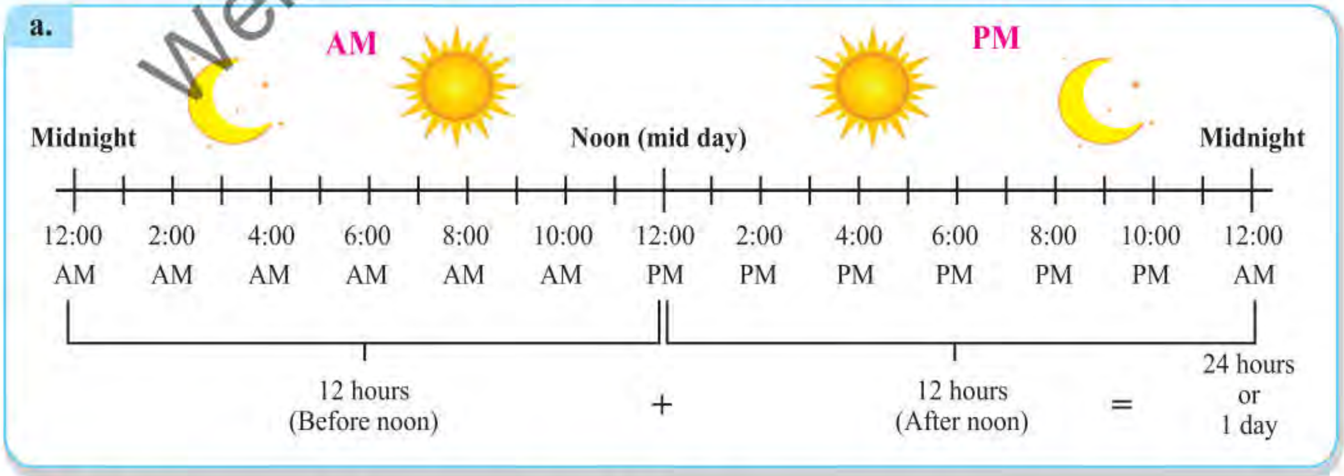
4.4 TIME

Let's observe two clocks.

24-hours clock

12-hours clock

As, we know that there are 24 hours in a day. To record time of the day we simply use 12 hours or 24 hours clock. The following number lines show few examples of different times of the day.



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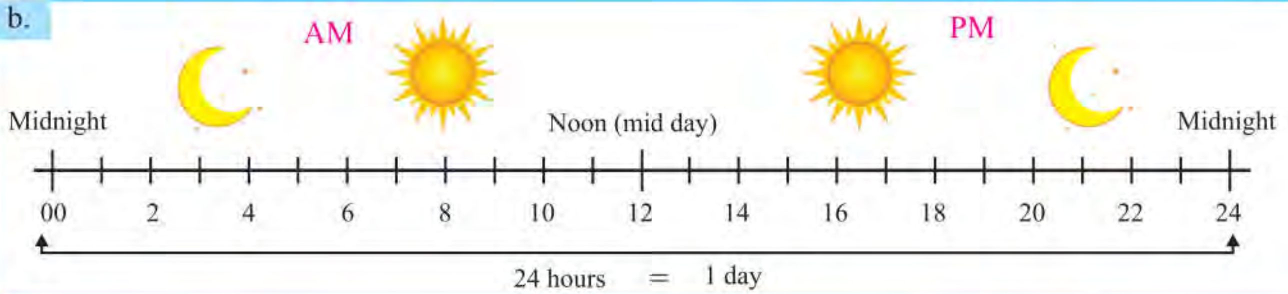
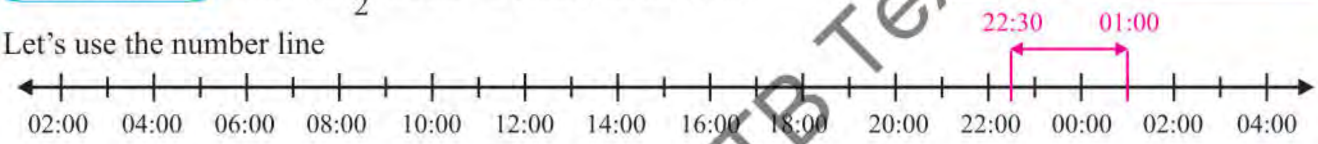
b.



Example 19 Waqas leaves Lahore at 22:30 on Sunday and arrives Pirmahal $2\frac{1}{2}$ hours later. At what time Waqas reached Pirmahal?

Solution Since, $2\frac{1}{2}$ hours = 2 hours, 30 minutes.

Let's use the number line



Hence, Waqas arrives Pirmahal at 01:00 or 01:00 a.m. on Monday.

Practice -6 Ajmal started playing cricket at 14:45 and ends at 18:00. How much time did he spend while playing cricket?

Remember

- a.m. stands for "ante meridiem" which means before mid day.
- p.m. stands for "post meridiem" which means "after mid day".

Activity -5 Record the today's morning assembly time at your school on your notebook. Also, record your arrival time at home from school. Calculate the time interval between your recorded times.

Exercise - 4.4

1. Convert the following 12-hours clock time into 24-hours clock time:

(i) 7:45 p.m. **(ii)** 9:31 a.m. **(iii)** 8:35 p.m.

2. Convert the following 24-hours clock time into 12-hours clock time:

(i) 15:48 **(ii)** 23:15 **(iii)** 19:43

3. A bus leaves for Peshawar at 4:30 p.m. It takes 6 hours to reach there. What time will it reach in Peshawar?

4. Nadia sleeps for 6 hours 30 minutes in a day. Today if she sleeps at 22:15, then on what time will she wake up?

5. The arrival time of Allama Iqbal Experess at Lahore junction is 11:40 a.m. and its departure time is 12:10 p.m. How much time Alama Iqbal Express stayed at Lahore junction?

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Project - 4 Make your daily schedule time table on a chart paper in both 12 hours clock and 24 hours clock. Also, calculate the time intervals between each salah time.

4.5 RELATION BETWEEN DISTANCE, TIME AND SPEED

We plan to travel, we always choose an appropriate mean which takes less time to cover the more distance. Time and distance are related to each other with speed.

For example: Zurain is travelling from Lahore to Bahawalpur. The distance covered by Zurain's car from Lahore to Bahawalpur at various intervals of time is shown below.

Time	**1*h***	**2*h***	**3*h***	**4*h***	**5*h***
Distance covered	86*km*	172*km*	258*km*	344*km*	430*km*

As more time taken covers the more distance. Now, look at the following table that shows the time taken by Zurain's car to travel from Lahore to Bahawalpur at various speed.

Speed of car	**43*km/h***	**86*km/h***	**129*km/h***
Time taken	10*h*	5*h*	3*h*

As more the speed of car takes the less time to cover the distance from Lahore to Bahawalpur.

When an object moves in a straight line at a speed, we can calculate its speed if we know how far it travels and how long it takes. This equation shows the relation between speed, distance travelled and time taken. i.e.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

The unit of speed is *km/h* (it is common unit).

Example 20 Nadeem covers a distance of 150 *km* on his motorcycle in 3 hours. Calculate the speed of his motorcycle.

Solution

Distance covered by Nadeem's motorcycle = 150 *km*
Time taken = 3 hours
Speed of motorcycle = ?

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{150\ km}{3\ h} = 50\ km/h$$

Hence, the speed of Nadeem's motorcycle is 50 *km/h*.

4.5.1 Conversion of Unit of Speed (Kilometre per Hour, Metre per Second and Vice Versa)

The most common unit of speed is kilometre/hour or miles per hour. These are the units that speedometer of your vehicle will show. There are also few other units of speed such as metre per second and kilometre per second etc.

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To convert the kilometre per hour to metre per second, we use

$$\frac{\text{Kilometre}}{\text{Hour}} = \frac{1\ km}{1\ \text{hour}} = \frac{1 \times 1000\ m}{1 \times (60 \times 60)\ sec}$$

$$= \frac{1000\ m}{3600\ sec} = \frac{5}{18}\ m/sec$$

Remember

1 *km*	=	1000 *m*
1 *m*	=	100 *cm*
1 hour	=	60 minutes
1 minute	=	60 seconds
1 hour	=	3600 seconds

So, we can convert *km/h* into *m/sec* by multiplying $\frac{5}{18}$ *m/sec*.

Example 21

Convert 100 *km/h* into *m/sec*.

Solution 100 *km/h*

$= 100 \times \frac{5}{18}\ m/sec$

$= 27.78\ m/sec$

Example 22 The speed of car is 150 *km/hour*. Find the speed of car in metre per second.

Solution The speed of car = 150 *km/hour*.

$= 150 \times \frac{5}{18}\ m/sec$ $\therefore\ 1\ km/h = \frac{5}{18}\ m/sec$

$= 41.7\ m/sec$

To convert metre per second to kilometre per hour we use

$$\frac{\text{Metre}}{\text{Second}} = \frac{1\ m}{1\ sec} = \frac{\left(\frac{1}{1000}\ km\right)}{\left(\frac{1}{60 \times 60}\ h\right)} = \left(\frac{60 \times 60}{1000}\right)\frac{km}{h}$$

$$= \frac{18}{5}\ km/h$$

So, we can convert *m/sec* into *km/h* just by multiplying $\frac{18}{5}$ *km/h*.
Now, look at the following examples:

Example 23 Saba's car covers a distance of 2000 *m* in 5 minutes. Calculate the speed of her car in *km/h*.

Solution

Distance covered by Saba's car	= 2000 *m*
Time take to cover the distance of 2000 *m*	= 5 minutes
Speed of car	= ?

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$= \frac{2000\ m}{5\ \text{minutes}}$

$= \frac{2000\ m}{5 \times 60\ sec}$

$= \frac{2000\ m}{300\ sec}$

$= 6.7\ m/sec$ $\therefore\ 1\ m/sec = \frac{18}{5}\ km/h$

$= 6.7 \times \frac{18}{5}\ km/h$

$= 24.12\ km/h$

Hence, the speed of Saba's car is 24.12 *km/h*.

Practice -7

- Express 45 *m/s* in *km/h*.
- Express 150 *km/h* in *m/s*.

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Exercise - 4.5

1. Convert the following into *m/sec*:

(i) 14 *km/h* **(ii)** $40\frac{1}{2}$ *km/h* **(iii)** 250 *km/h*

2. Convert the following into *km/h*:

(i) 25 *m/sec* **(ii)** 130 *m/sec* **(iii)** $340\frac{1}{2}$ *m/sec*

3. A car travels a distance of 749 *km* in 7 hours. What was the speed of car?

4. Neelam cycles for 4 hours at a speed of 40 *km/h*. How much distance she covered?

5. A bus travels from one station to the other station, a distance of 672 *km* in 7 hours. The bus is only allowed to travel at 95 *km/h*. Did the bus break the speed limit?

6. Shumaila is driving across the country. For the first 4 hours, she travels at 80 *km/h*. For the next 3 hours, she travels 90 *km/h*. Assuming that she has not stopped, what is her average travelling speed in *km/h*?

Project - 5 Suppose you are late from your school today. You have only 30 minutes to reach your school. How much fast you need to travel to reach at school on time?

Review Exercise 4

1. Each of the question below is followed by four suggested options. Choose the one option that is correct.

(i) The first term / element in ratio is called:

a. consequent b. antecedent c. denominator d. numerator

(ii) A ratio is written by putting the sign_____:

a. ; b. : : c. : d. "

(iii) The second term in the ratio is called:

a. consequent b. antecedent c. denominator d. numerator

(iv) The ratio between the length and breadth of our national flag is:

a. 3 : 2 b. 2 : 3 c. 1 : 2 d. 3 : 1

(v) The ratio has no.

a. symbol b. value c. importance d. unit

(vi) The symbol of percent is:

a. ÷ b. % c. × d. +

(vii) The word "per – centum" meaning is out of:

a. 10 b. 100 c. 1000 d. 10,000

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(viii) In percentage related problem the word 'of' means:

a. addition or sum
b. subtraction or difference
c. times or multiplication
d. division or distribution

(ix) The percentage of given quantity can be calculated by using the formula:

a. $percentage = \frac{whole\ quantity}{part\ of\ quantity} \times 100\%$
b. $percentage = \frac{part\ of\ quantity}{whole\ quantity} \times 100\%$
c. $percentage = \frac{part\ of\ quantity}{whole\ quantity} \times \frac{1}{100}\%$
d. $percentage = \frac{total\ quantity}{obtained\ marks} \times 10\%$

(x) 12 out of 15 can be written in percentage as:

a. 70% b. 80% c. 85% d. 90%

(xi) 130% of 200 is:

a. 210 b. 230 c. 250 d. 260

(xii) An increase of 20% in Rs. 500 is:

a. 50 b. 100 c. 150 d. 200

(xiii) A decrease of 7% in Rs. 1200 is:

a. Rs. 70 b. Rs. 74 c. Rs. 84 d. Rs. 90

2. Give the short answers for the following:

(i) Define ratio. (ii) What do you know about antecedent and consequent? (iii) Define rate. (iv) What is the difference between rate and ratio? (v) Define percentage. (vi) What is 14% of 50? (vii) Define continued ratio.

3. Write the following ratios in the simplest form:

(i) 90:100 (ii) 138 to 414 (iii) 13:21 (iv) 84:96:108 (v) 58:87:116 (vi) 124 to 155 to 186

4. Distribute Rs. 19740 among three sisters Rabia, Saima and Alina in a way, Rabia : Saima = 3:5 and Saima : Alina = 5:7

5. Nadeem can type 2000 words in one hour. How many words he can type in 10 minutes?

6. Mushtaq spent Rs. 350 out of Rs. 1000. What percent of amount did he spend?

7. In a school election for head boy. Aleem got 52% of the votes and Hamza got the remaining votes. If the total number of vote cast is 500, then find the votes obtained by Hamza.

8. Aleezey bought a doll for Rs. 1500 after getting a decrease of 30%. Find the original price of doll.

9. The result of a number is 200. If it is decreased by 20%, then what is the original number?

10. Nadeem is a shopkeeper. He has 300 oranges at his shop that he has 15% more bananas than oranges and 30% less oranges than apples. Find the number of bananas and apples on the shop.

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11. There are 36 boxes of fruit drink in 6 packets. How many drink have there in 10 packets?

12. Irza's cat catches 9 mice every 3 days. Laiba's cat catches 21 mice every 7 days. Do the two cats catch mice at the same ratio?

13. The rent of a house for 3 months is Rs. 54000. How much rent is paid for 5 months?

14. A train takes 2 hours to cover 150 kilometre. How long will it take the train to cover 900 kilometre?

Go Online Visit the link to play online game about the topics discussed in this unit
https://www.brainpop.com/games/unitrates/?topic_id=

Summary

- The ratio is a comparison between two numbers or things of same kinds.
- Ratio can be written in three forms:
  (i) Word form (ii) Ratio form (iii) Fraction form
- Ratio is unit less or it has no unit.
- The first term in ratio is called antecedent while the second term is called consequent.
- A rate is a special kind of ratio, in which the two terms are in different units.
- Any ratio having second term is 100 is called percentage.
- We use the symbol "%" instead of writing the word percentage.
- Any fraction can be written as percentage by multiplying $\frac{100}{100}$ or 100%
- For increasing or decreasing of a quantity in a given percentage, generally we use the formulae:
  (i) New price = Final percentage × original price
  (ii) Increase/decrease = percentage increase/decrease × original value

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Challenge

- Saba has 72 bottle caps.
- 50% of them are green.
- 0.25 of all caps are blue.
- They are in red, blue, green and yellow.
- One third of all caps are red.
- The remaining caps are yellow.
- How many bottle caps are there of each colour?

Teaching Point

The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

Sub-Domain 5

SETS

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Use language, notation and Venn diagrams to represent different types of sets and their elements (empty, singleton, subsets, proper, improper subsets and universal set, finite and infinite sets).

Can you form group of coins using these coins? If yes! then how many groups can be formed?

INTRODUCTION

In our daily life we often use some words to speak which means "a collection" or "a grouping". For example,

Pile of books

Bunch of keys

School uniform a student wears

Pakistan cricket team

In Mathematics we use the word "set" to describe any collection of objects.

"A set is a collection of well-defined and distinct objects."

The word well-defined means clearly described. There should be no uncertainty about which object belongs to the set and which does not. The above examples are the examples of set because the elements in each collection are well-defined and distinct, whereas the word "distinct" means no two objects of a set are identical.

Consider the following examples which are not representing the set:

(i) All good teachers in Punjab who participated in the contest.

(ii) All good storybooks in the library.

Here, we can examine that the word 'good' is not well define because a teacher may be good view of one person but may not be good for other. Similarly, a storybook in the library may be good in view of one person but may not be good for others.

The above example become sets if we rewrite them as

(iii) All teachers of Mathematics in Punjab who participated in the contest. (Here the word "Mathematics" makes the statement true for a set).

(iv) All English storybooks in the library. (Here the word "English" makes the statement true for a set).

History

George Cantor (1854-1981) was a Mathematician of 19th century. He was the first person who gave the concept of set theory. Which became the fundamental theory in Mathematics. To know more about him, visit the link:

https://www.britannica.com/biography/Georg-Ferdinand-Ludwig-Philipp-cantor

Here in this unit we will learn in detail about sets, its notations and different types etc.

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5.1 NOTATION OF A SET AND ITS OBJECTS/ELEMENTS

Notation is a system of marks, signs, figures or characters that is used to represent information. Generally a set is represented with capital letter of English alphabet i.e. *A, B, C,* ..., and *Z*. But the elements of a set are written in brackets { }, and separated by comma.

For example,

A = Set of days of a week.

The symbol '∈' is used to show the membership of a set and read as *"belongs to"* but '∉' is used for *"does not belong to"* a set.

For example,

Monday ∈ A but March ∉ A

Friday ∈ A but {Friday} ∉ A

Remember

Sunday ∈ B but {Sunday} ∉ B, because 'Sunday' is an element of set B and {Sunday} is another set.

Although any capital letter can be used for a set but some specific sets are denoted by certain letters, which are:

N = The set of natural numbers = {1, 2, 3, ...}

W = The set of whole numbers = {0, 1, 2, ...}

Z = The set of integers = {..., –2, –1, 0, +1, +2, ...}

E = The set of even numbers = {0, 2, 4, 6, ...}

O = The set of odd numbers = {1, 3, 5, 7, ...}

P = The set of prime numbers = {2, 3, 5, 7, ...}

C = The set of composite numbers = {4, 6, 8, 9, ...}

Practice -1 If A = {3,4,7,8,9}, then put the correct sign ∈ or ∉ in the blank space.

(i) 2_______A **(ii)** 9_______A **(iii)** 13_______A **(iv)** 8_______A

Note

The curly bracket { } are some time called set brackets or braces.

Example 1 Write the elements of the following sets using symbol ∈:

(i) A = Set of first five even numbers. **(ii)** B = Set of all provinces of Pakistan.

(i) First five even numbers are 0, 2, 4, 6, and 8. These are elements of set A. We can write them as:

0 ∈ A
2 ∈ A
4 ∈ A
6 ∈ A
8 ∈ A

(ii) All provinces of Pakistan are Punjab, Khyber Pakhtunkhwa, Sindh and Balochistan. These are all elements of set B. We can write them as:

Punjab ∈ B
Khyber Pakhtunkhwa ∈ B
Sindh ∈ B
Balochistan ∈ B

Teaching Point Perform different activities with students in the class to clear the concept of sets. e.g.

- divide students into groups of 5-7.
- give one set of numbers to each group.
- write the name of students on the board.
- call out a problem such as "the cardinal number of set A" where, total number of elements of set A is its cardinal number.
- the first group to raise the correct number card reflecting the answer to your question gets a point.
- continue for as long as desired. The group with the most points at the end of the game wins.

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5.2 DESCRIBING A SET

Mostly a set can be described in the following forms:

Descriptive form

In descriptive form of a set, we simply describe a set in words. For example:

A = Set of items of school uniform a student wears.

B = Set of first five positive odd natural numbers.

Remember

The dots (…) means continue on:
The (ellipsis) between the first three elements and the last element denote the set is finite.
But the dots at the end shows that set is infinite.

Tabular form

In tabular form of a set we list each element (or number), separated by a comma and then put curly brackets around the whole things. This form of separation is known as tabular form. For example:

A = {shoes, pent, socks, shirt, tie}

B = {1, 2, 3, …, 10}

X = {Punjab, Sindh, Khyber Pakhtunkhwa, Balochistan, Gilgit Baltistan}

Note The descriptive form of a set is also known as statement form.

Example 2 Write the following sets in tabular form:

(i) A = Set of colours in rainbow.

(ii) B = Set of vowels of English alphabet.

Solution

(i) A = Set of colours in rainbow.

Tabular form: A = {red, orange, yellow, green, indigo, violet, blue}

(ii) B = Set of vowels of English alphabet.

Tabular form: B = {a, e, i, o, u }

Example 3 Write the following sets in descriptive form.

X = {blue, yellow, red }

Solution X = {blue, yellow, red }

Descriptive form: X = Set of primary colours.

Note

The tabular form of a set is also known as roster form.

Just for Information

- In light the primary colours are red, green and blue. Write all colours in descriptive form of set.
- In pigment the primary colours are red, yellow and blue. Write all colours in tabular form of set.
- In physics the primary colours are slightly different from red, green and blue. Write all colours in tabular and descriptive form.

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Exercise - 5.1

1. State which of the following group of things is set and which is not? Also give the reason:

(i)
(ii)
(iii)
(iv)

2. Which of the following statements represent set. If yes then write 'true' in the given space otherwise write false:

(i) Set of all good pupils in Multan. (________)
(ii) Clever students in your school. (________)
(iii) Set of all prime numbers between 10 and 50. (________)
(iv) Set of all alphabet in the word Mathematics. (________)
(v) Set of all workbooks in your school bag. (________)
(vi) Set of reptiles. (________)
(vii) A = {1, 2, 3, 3, 4} (________)
(viii) X = {Waqas, Ali, Waqas}. (________)
(ix) Set of Nobel prize winners from the Punjab. (________)

3. If P = Set of prime numbers = {2, 3, 5, …} then fill in the blanks using the symbols $\in$ or $\notin$:

(i) 3 _____ P **(ii)** 14 _____ P **(iii)** 7 _____ P **(iv)** 9 _____ P **(v)** 39 _____ P
(vi) 29 _____ P **(vii)** 16 _____ P **(viii)** 13 _____ P **(ix)** 43 _____ P

4. Write the following sets in tabular form:

(i) A = Set of seasons in a year. **(ii)** B = Set of names of days starting with W.
(iii) C = Set of Mathematics teachers in your school. **(iv)** D = Set of your siblings.
(v) E = Set of letters in the word Punjab. **(vi)** F = Set of last six months of Islamic calendar.
(vii) G = Set of last four prime ministers of Pakistan.
(viii) H = Set of first four prime numbers.
(ix) I = Set of all vowels used in the sentence ORANGE TRAIN.

5. Write the following sets in descriptive form of sets:

(i) A = {0, 1, 2, 3, 4, 5 } **(ii)** B = {Football, Hockey, Badminton, Cricket}
(iii) C = {a, b, c, d, …, z} **(iv)** D = {Lahore, Karachi, Peshawer, Quetta}
(v) E = {July, August, September, October, November, December }
(vi) F = {21, 22, 23, 24, 25, 26, 27, 28, 29, 30}
(vii) G = {Punjab, Sindh, Khyber Pakhtunkhwa, Balochistan}
(viii) D = {0, 2, 4, 6, 8, 10, 12, 14 } **(ix)** I = {Orange, Banana, Mango}

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Project - 1 Use different objects at your home and form atleast 3 different sets. Name them as *A*, *B* and *C*. On the next day show your working to your teacher.

5.3 DIFFERENT TYPES OF SETS

Thinking Time

Why empty set is a set?
Even that there is no element in it.

Following are the different types of sets:

Empty / Void / Null set

We know very well about the meaning of the word empty. It means containing nothing, for example, when there is no student in the class. It means the class is empty. Similarly, when there is no element in the set, it means the set is empty. So, an empty set can be defined as,

"A set which does not contain any element is known as empty set or void set or null set".

It is denoted by { } or ϕ, we read ϕ as phi,

For example, B = The integers between 5 and 6

B = { } or B = ϕ

Note

{ϕ} or {{}} are not empty sets, because each contains one element namely the empty set ϕ or { } itself.

Singleton set

A set containing only one element is known as singleton set.

For example, A = set of even prime numbers

B = {2}, C = $\left\{\frac{1}{4}\right\}$

Equal and equivalent sets

(i) Equal Sets: Two sets A and B are said to be equal sets if they have the same elements. Every element of set A is an element of set B. Similarly, every element of set B is an element of set A. It does not matter what order the elements are in the set, it just matter that the same elements are in each set. The equal sets are denoted as A = B.

For example, A = {1, 2, 3, 4, 5 } , B = {1, 2, 3, 4, 5 }
So, A = B

P = {a, e, i, o, u } , Q = {a, e, u, i, o }
So, P = Q

(ii) Equivalent Sets: Two sets A and B are said to be equivalent sets if they have the same number of elements. It is not necessary that the members of both sets are same. For example,

A = {2, 3, 4, 5} , B = {c, d, e, f}

Number of elements in set A = 4 , Number of elements in set B = 4

As, the number of elements of set A and B are same so, A is equivalent to B. It is denoted by A $\leftrightarrow$ B, '$\leftrightarrow$' is the symbol which we use to denote the equivalent sets. Sometimes we use '~' for equivalent relation.

In general: Two or more sets are equivalent sets if their cardinal numbers are same, e.g.

$n(A) = 3$ and $n(B) = 3$ so, $n(A) = n(B) \Rightarrow A \leftrightarrow B$

Teaching Point Us the below online video link to explain the different types of sets.
http://www.youtube.com/watch?v=8dup8yGWBhM

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Remember The number of elements in a set is called its cardinal number. For example, A = {a, b, c, d} The cardinal number of A is n(A) = 4. So, For equivalent sets the cardinal number should be same.

Note Two or more equal sets are also equivalent sets, but equivalent sets may or may not be equal. Infinite sets are always equivalent sets.

Challenge If X and Y are two non-empty sets and n(A) = n(B) then explain A = B or A ≠ B, explain with a counter example.

Subset and superset of a set

Remember To find the possible number of subsets we simply use 2^n.

(i) Subset

"A subset is a set which is contained in another set".

In other words, a set A is said to be a subset of the set B if all the elements of set A are the elements of set B. It is denoted by the symbol $\subseteq$.

For example, A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5}

As every element of set A is an element of set B so, A is a subset of B. Symbolically it is written as $A \subseteq B$. If A is not subset of B then we write $A \nsubseteq B$.

Remember
- Every set is a subset of itself. i.e. $A \subseteq A$, $B \subseteq B$
- Empty set or ϕ is always a subset of every given set.
- If $A \subseteq B$ and $B \subseteq A$ then A = B

(ii) Superset

"A superset is set which contains another set completely" e.g.

If $A \subseteq B$, then set B is called the superset of set A. In the above example, B is superset of set A. Symbolically, it is written as $B \supseteq A$.

Proper and improper subsets of a set

(i) Proper subset

"A proper subset of a set A is a subset of set A that is not equal to set A".

In other words, if B is proper subset of A then all elements of B are in A but A contains atleast one element that is not in B. Symbolically it can be written as $B \subset A$.

For example,

A = {a, e, i, o, u}
B = {a, i, o, u}
$B \subset A$ because $e \notin B$ but $e \in A$

Thinking Time
- Is an improper subset a superset?
- Are equal sets supersets of each other?

(ii) Improper subset

Improper subset is a set which contains all the elements of original set.

For example, A = Set of natural numbers less than 6.
B = Set of first five natural numbers.
As, B = A So, B is an improper subset of A and we write it as $B \nsubseteq A$.

Go Online
Use the following online game link for practice of symbols used in set.
https://www.transum.org/software/SW/Starter oftheday/Students/SetNotation/Default.asp

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Activity -2 Choose any three students from your class. Write their names in a set. Then make all subsets and separate them as proper and improper subsets.

Practice -2 Write all the subsets of A = {p,q} and separate them as proper and improper subsets.

Universal set

Universal means pertaining to the whole of something specified, occurring everywhere. In sets, "A universal set is a collection of all elements or members of all related sets, known as its subsets".

Mostly a universal set is denoted by 'U' . For example, your school building can be considered as universal set and different classes e.g. primary section classes are elements or members of universal set. In other words.

U = All classes and teaching staff in your school.
A = Primary section classes.
B= Middle section classes.

Remember
A universal set can be a finite or infinite set.

When we study numbers in Mathematics we are interested in the set of natural numbers. The basic set is considered as a universal set, its subsets are, even numbers and odd numbers.

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 4, 6, 8, 10}, B ={1, 3, 5, 7, 9}

Finite set and infinite set

(i) Finite set

" A set consisting of a fixed number of elements which are countable, is called finite set". For example,

A = Number of students in grade 6, B = {8, 9, 10, 15, 20}

(ii) Infinite set

"A set consisting of unlimited number of elements which are not countable, is called an infinite set". For example, A = Stars on the sky, B = set of whole numbers, C = {1, 3, 5, 7, ...}

Practice -3 Separate the finit and infinite sets.
A = Students of grade-6, whose name start with alphabet 'T', B = {1, 3, 5, 7, 9, 11, ...}, C = {0, 2, 4, 6, 8, ..., 100}.

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Exercise - 5.2

1. Separate the empty set from the following sets:

(i) Set of even numbers between 6 and 7.
(ii) Set of integers between 0 and 1.
(iii) Set of hair on your head.
(iv) Set of English alphabet between P and Q.

2. Separate the finite and infinite sets. First one is done for you:

(i) Set of all multiples of 1 less than forty. (Finite)
(ii) Set of all classes in your school.
(iii) Set of all players in Pakistan cricket team.
(iv) Set of natural numbers.
(v) Set of all girls in the school.
(vi) {10, 20, 30, …,80}
(vii) Set of all Government High Schools in the Punjab.
(viii) Set of odd numbers.
(ix) Set of natural numbers greater than 30.

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3. Which of the following are the singleton sets? Also write the cardinal number of each set:

(i) A = {4, 5, 7} **(ii)** B = {Waseem} **(iii)** C = {a, b, c, d, e}

(iv) D = Set of odd numbers between 8 and 10. **(v)** E = Set of composite numbers between 0 and 11.

(vi) F = Set of even prime numbers less than 10.

4. If A = Set of whole numbers up to 11, B = {2,1, 3},C = { }, D = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, E = {2}, F = {3, 4, 5}. Then put the correct symbol ($\subseteq$, $\supseteq$, $\subset$, $\not\subset$, or $\longleftrightarrow$).

(i) A _____ B **(ii)** D _____ A **(iii)** C _____ E **(iv)** F _____ A **(v)** B _____ F

(vi) E _____ B **(vii)** D _____ B **(viii)** F _____ C **(ix)** E _____ E

5. Identify which of the following pair of sets are equal or equivalent?

(i) A = {Waqas, Waseem, Asghar}, B = {Asghar, Waseem, Waqas}

(ii) V = {a, e, i, o, u}, W = {u, i, o, e, a}

(iii) X = {1, 2, 3, 4, 5, 6}, Y = {1, 2, 3, 4, 5, 6}

(iv) P = {pq, rs, t}, Q = {pq, rs, t}

(v) D = Set of vowels, E = {a, b, c, d, e}

(vi) L = Set of even numbers between 5 and 8

M = Set of whole numbers between 4 and 6

(vii) N = {hockey, football, cricket} 0 = {cricket, hockey, football}

6. Write all the subsets of A = Set of primary colours, then identify the proper subsets and improper subsets.

7. Show that the following sets are equal.

P = Set of prime numbers less than 15, Q = {2, 3, 5, 7, 11, 13}

8. A = Set of letters in the word Faisalabad. Write the cardinal number of set A.

9. Write the name of your all family members in set notation then write a subset of your family members.

Project - 2 Make a booklet for all the different types of sets. Explain all types by using real life examples from your surroundings.

5.3 VENN DIAGRAM

We can represent all possible logical relationships between a finite collection of sets pictorially by using Venn diagram. In Venn diagram the elements of sets are enclosed within closed curve (circle or oval) and the universal set represented by a rectangle as shown in the figure 5.1

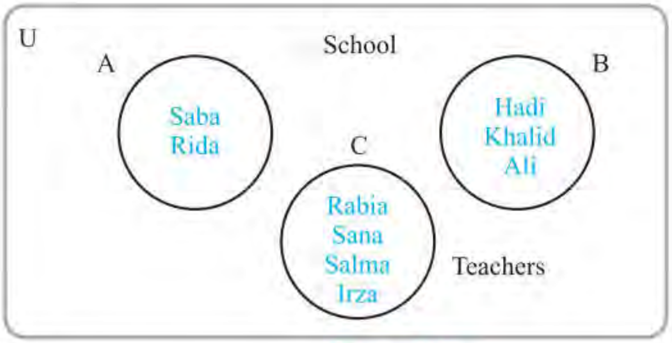

Figure 5.1

where, U = All students of grade (4-6) and teaching staff

Grade-4 = {Saba, Rida} Grade-5 = {Hadi, Khalid, Ali} Grade-6 = {Rabia, Sana, Salma, Irza}

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History

John Venn was an English logician and mathematician. He introduced the venn diagram first time in 1880. To know more about him, visit the link: https://www.britannica.com/biography/John-venn

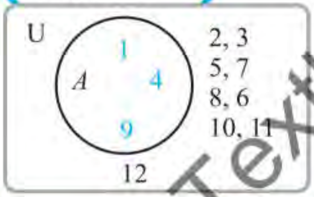
5.3.1 Representation of Sets Through Venn Diagram

It is very easy to understand relationship if we have something in visuals. Look at the following examples:

Example 5

If U = {1, 2, 3, 4, 5, ..., 12} and A = {1, 4, 9} then draw a venn diagram to represent the sets U and A. Also, write the cardinal number of each set.

Solution



Now, the cardinal numbers.
n(U) = 12 and n(A) = 3

Practice -4

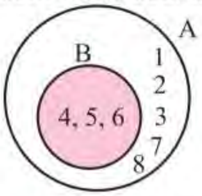
If U = all colours of rainbow and X = {red, green, blue} then represent these sets using Venn diagram.

5.3.2 Representation of Subsets Using Venn Diagram

Example 6 If A = {1, 2, 3, 4, 5, 6, 7, 8} and B = {4, 5, 6} then represent these sets using Venn diagram.

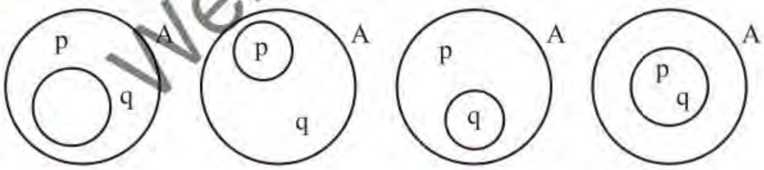
Solution

As, B ⊆ A then we can draw the Venn diagram as shown.



Example 7 If A = {p, q} the list of all proper subsets, improper subsets and draw Venn diagrams of these sets.

Solution Look at the below figures.



(i) (ii) (iii) (iv)

Practice -5

Write all the subsets of X = {1, 2, 3} and represent them using Venn diagram.

Since, A = {p, q}

Proper subsets: ϕ, {p}, {q} and Improper subset: {p, q}

Teaching Point Make three sets of hobbies. e.g. sports, reading and computer games.

- Set A for people who like sports.
- Set B for people who like reading.
- Set C for people who like computer games.

Write the names of students in each set and construct Venn diagram.

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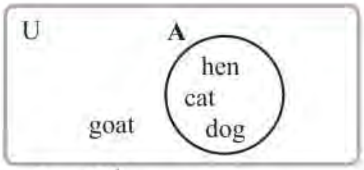
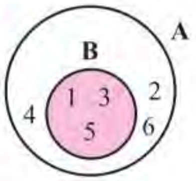
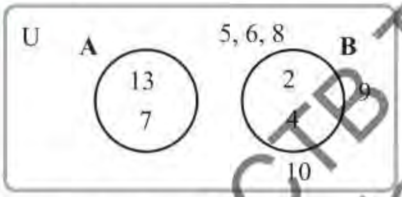
Go Online Use the below link to practice online for venn diagram puzzles.
https://www.vnivie.ac.at/future.media/moe/tests/mengen/duv.html

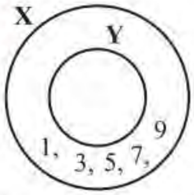
Exercise - 5.3



1. The Venn diagram shows the elements in sets U and A then list all the elements of set U and A respectively.
2. It is given that U ={a, c, d, e, f, i, g, o, h, u}, X = {a, e, i, o, u} and Y = {c, d}. Draw a Venn diagram to show the above information.
3. It is given that U = Set of natural numbers less than 15, A = Set of natural numbers between 6 and 11 and B = Set of even prime numbers. Draw a Venn diagram to show the above information.
4. Read the Venn diagram and list the elements of each set.

(i)


(ii)


(iii)


5. It is given that A = {Nadeem, Neelam, Noureen, Nimra} and B = {Neelam, Noureen} then draw a Venn diagram to represent the sets A and B.

Project - 3

List all the name of your family members in a set. Then write all the subsets of that set. Finally draw a Venn diagram of each subset.
Note: Write maximum 4 names of your family members.

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Review Exercise 5

1. Each of the following question is followed by four suggested options. In each case select the correct option.

(i) A set is always denoted by:
   a. Capital letters of English alphabet b. Roman Numerals
   c. Small letters of English alphabet d. Arabic numerals

(ii) Statement form of set is also known as:
   a. Tabular form b. Descriptive form c. Roster form d. Set Builder form

(iii) Roster form of sets is also known as:
   a. Tabular b. Descriptive c. Set-builder d. statement

(iv) Symbols used to represent empty / void / null set are:
   a. $=$ or $\cong$ b. $\subset$ or $\subseteq$ c. $\{\ \}$ or ϕ d. $\in$ or $\notin$

(v) Any set containing only one element is called:

a. Even set b. Singleton set c. Positive set d. Odd set

(vi) To find the total number of subsets of any given set we use:

a. 2n b. n_2 c. n^2 d. 2^n

(vii) A set containing no element is known as:

a. Universal set b. Singleton set c. Empty set d. Subset

(viii) The cardinal numbers of set A = {Pakistan, China, Bangladesh}:

a. 1 b. 3 c. 2 d. 4

(ix) If A = Set of letters in the word Islamabad then:

a. $x \in A$ b. $m \in A$ c. $c \in A$ d. $f \in A$

(x) The universal set in the Venn diagram is represented by the shape:

a. b. c. d.

2. Define: • Finite set • Infinite set • Universal set

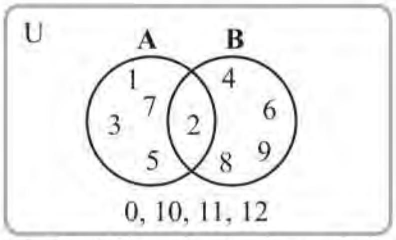
3. Write the following sets in another form of sets:

(i) A = Set of your five friends (ii) B = {0, –1, –2, –3, ...}

(iii) Set of school diaries in your bag

4. Write all the subsets of A = {1, 2, 8}.

5. Observe the Venn diagram and list all the elements of each set.



6. If U = Set of integer between 1 and 30, A = {3, 6, 9, 12, 15, 18, 21, 27} and B = Set of all factors of 30 then draw a Venn diagram to illustrate the given information.

Challenge

If A = {1, {2}} then write all subsets of A.

Summary

- A set is a collection of well defined and distinct objects.
- A set having a fixed number of countable elements is known as finite set.
- A set having unlimited number of uncountable elements is known as infinite set.
- A set which does not contain any element is known as empty / void / null set.
- A set containing only one element is known as singleton set.
- An improper subset is a subset which contains all the elements of original set.
- A universal set is a collection of all elements or numbers of all related sets known as its subsets.
- A Venn diagram is used to show the relationships between a finite collection of sets pictorially.

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Domain | Algebra

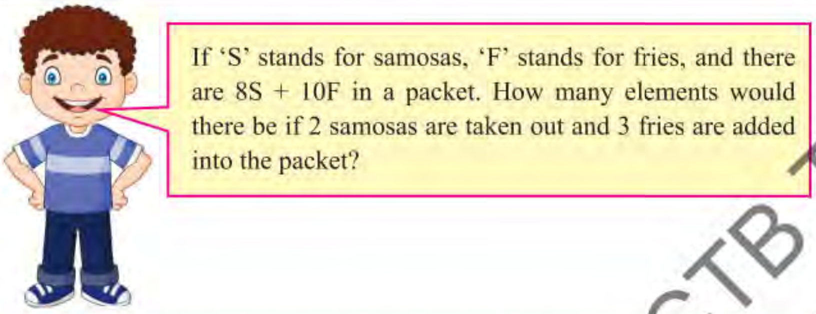
Sub-Domain 6

ALGEBRAIC EXPRESSIONS

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Recognize simple patterns from various number sequences.
- Use letters to represent numbers, express basic arithmetical processes algebraically.
- Evaluate algebraic expressions, add and subtract linear expressions.
- Simplify linear expressions.



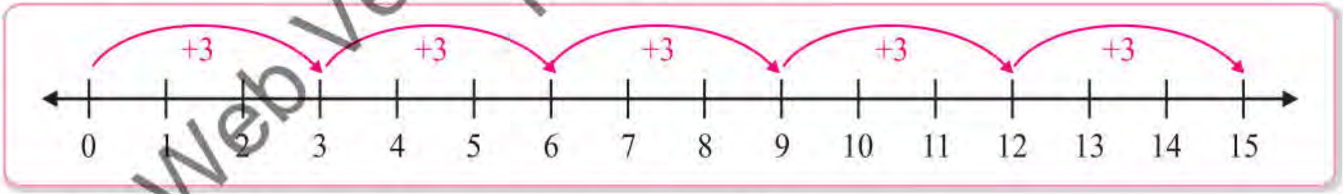
6.1 RECOGNITION OF SIMPLE PATTERNS FROM VARIOUS SEQUENCES

We know that "A pattern is some phenomenon that repeats regularly based on a set rule or condition." In our previous classes we have learnt about different patterns.

6.1.1 Number Pattern/Sequence

Consider the following whole numbers.

0, 3, 6, 9, 12, 15, ...



Here, we get the numbers in the pattern/sequence by adding 3.

To solve the problems related to number patterns, we first need to find the rule followed in the given pattern/sequence, see the difference in the consecutive numbers in the given sequence. It will help you to understand the relationship between the numbers.

Teaching Point

Encourage students to ask questions at any stage during learning process. Students questioning serve a wide variety of purposes. e.g.

- to assess their understanding.
- to keep learners engage during an explanation.
- to develop their thinking or focus their attention on topic.

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Example 1 For each of the following number pattern state a rule and find the next two terms:

(i) 17, 22, 27, ... **(ii)** 56, 53, 50, ...
(iii) 729, 243, 81, ... **(iv)** 4, 8, 12, ...

Remember
Each number in the given number sequence is called term.

Solution

(i) 17, 22, 27, ...

Rule Add 5 in the each term to get the next term in the sequence/pattern. So, the next two terms will be 32 and 37.

(ii) 56, 53, 50, ...

Rule Subtract 3 from each term to get the next term in the sequence. So, the next two terms will be 47 and 44.

(iii) 729, 243, 81, ...

Rule Divide each term by 3 to get the next term in the sequence. So, the next two terms will be 27 and 9.

(iv) 4, 8, 16, ...

Rule Multiply each term by 2 to get the next term in the sequence. So, the next two terms will be 32 and 64.

Practice -2 For each of the following sequences, state the rule to find the next two terms.

i. 4, 14, 42, ... ii. −7, −14, −21, ...

Challenge Which number pattern is used in sunflower?

Go Online Visit the link and play the sequence game online.
http://www.mathframe.co.uk/en/resouces/resource/42/sequences

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Exercise - 6.1

1. For each of the following sequence state a rule and find the next two terms:

(i) 9, 13, 17, ... **(ii)** 277, 287, 297, ... **(iii)** 14, 21, 28, 35, ... **(iv)** −84, −82, −80, ...

2. Write the next three terms in the sequence:

(i) 1, 4, 9, ... **(ii)** 89, 87, 85, ... **(iii)** 12, 16, 20, ... **(iv)** 1, 1, 2, 3, 5, 8, ...

Project - 1 Make a poster of polygons in sequence according to the number of sides.

Teaching Point Ask students to arrange in an increasing or decreasing order to give practice on sequences. You can use snacks or cereal to make a sequence of any pattern.

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6.2 USE OF LETTERS TO REPRESENT NUMBERS

We are very familiar about the natural numbers 1, 2, 3, ... and the use of basic four operations (addition, subtraction, multiplication and division) in arithmetic. In algebra, letters replace the numbers.

History

A muslim Arab Mathematician Al – khawrzmi (780 – 850 AD) is founder of modern algebra. He was one of the first to study algebraic expression. The word algebra is derived from the word Al – Jabr. He wrote a book named Al – Jaber – Wa – Al – Muqabala in 820 AD. He is known as the father of algebra.

To know more about him, visit the link: http://www.famousscientists.org/muhammad-ibn-musa-al-Khwarizmi

In Arithmetic: Numerals such as 1, 2, 3, … etc are used, with each numeral having a fixed value.

(i) $5 + 3 = 8$ means that the sum of 5 and 3 is equal to 8.

(ii) $13 - 5 = 8$ means that the difference of 13 and 5 is 8.

In Algebra: A letter can symbolize any numeral, or any value assigned to it for example.

(i) $a + b = c$ means that the sum of two numbers are represented by a and b is equal to c.

(ii) $c - a = b$ means that the difference of two numbers are represented by c minus a equal to b.

If $a = 5$ and $b = 3$ then $c = 8$, If $a = 5$ and $c = 12$ then $b = 7$ i.e. $c - a = b$
$12 - 5 = 7$

6.2.1 Explanation for Variable and Constant

(i) Variable

We know that in algebra, both numerals and letters of alphabet $a, b, c, \ldots, z$ are used. These letters of alphabet do not have any fix value.

So, "*A variable is a symbol or letter of alphabet that represents a quantity the value of which is not known*".

(ii) Constant

A constant is a quantity whose value remains unchanged (fixed).

Example 2 Identify variable and constants in the following:

(i) $n + 8 = 7$ **(ii)** $n + m + 4$ **(iii)** $\ell - 14 = 34$

Solution

	Expression	Variable	Constants
(i)	$n + 8 = 7$	n	7, 8
(ii)	$m + n + 4$	m, n	4
(iii)	$\ell - 4 = 34$	ℓ	–4, 34

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Did You Know?

The number 9 is a mysterious number. It can be found in every ones birth date either it contains 9 or not. As, I was born on 22-04-2001. The number 9 is hidden in this date. To find it, let's rewrite birth date as 22042001.

- Now, rearrange the digits any order such as 20010422.
- Subtract smaller number from greater e.g.

$$\begin{array}{r} 22042001 \\ -20010422 \\ \hline 2031579 \end{array}$$

- Add the digits of your result. $2 + 0 + 3 + 1 + 5 + 7 + 9 = 27$
- Add $2 + 7 = 9$, similarly, find the number 9 in your birth date.

Exercise - 6.2

1. Write the following sentences symbolically:

(i) The sum of numbers n and 6 is 20. **(ii)** The difference of numbers n and 27 is 5.
(iii) Four times a number n is less than 7. **(iv)** Two times a number n is more than 25.

2. Which of the following statements are true and which are false?

(i) Sum of five and eight is twenty. **(ii)** Product of three and ten is thirty. **(iii)** $9 + 19 = 28$.
(iv) $6 + (8 - 2) = 0$. **(v)** $\frac{7}{8} + 3 > 16$.

3. Write the values of unknown which satisfy the following statements:

(i) $\square + 18 = 36$ **(ii)** $3 : 5 :: \square : 10$ **(iii)** 20% of $2000 = \square$
(iv) $\frac{2}{5} + \square = \frac{1}{10}$ **(v)** $\frac{7}{11} + \frac{1}{22} = \square$

Project - 2

Create atleast 5 algebraic sentences and separate variables and constants from your sentences.

4. Identify the constants and variables.

(i) $n + 9 = 0$ **(ii)** $7p + 2 = 9$ **(iii)** $\frac{1}{3}xy$ **(iv)** $7i + 4 = 9$ **(v)** $m + n + 8$ **(vi)** $9x + y - 37$

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6.3 ALGEBRAIC EXPRESSIONS

An algebraic expression is an expression built up by the combination of constants, and variable by the basic algebraic operations (+, –, ×, ÷) for example $x + y$ and $4x + 5y + 5$ are algebraic expressions.

Example 3 Write the following statements in an algebraic expression:

(i) The sum of an integer and 17 **(ii)** The difference of 15 and double of a number n.
(iii) Product of a and $\frac{1}{2b}$ **(iv)** Divide p by $4q$

Solution

(i) Let the integer be 'n'
So, $n + 17$

(ii) Let the required number be 'n'
So, $15 - 2n$

(iii) $a \times \frac{1}{2b}$

(iv) $p \div 4q$

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Example 4 Alishah is x years old. Write an algebraic expression for the following.

(i) Alishah's age before six years.
(ii) Six times Alishah's age after five years.
(iii) Half of Alishah's age six years ago.
(iv) The present age of Alishah's younger sister if her age was 5 years less than the half of Alishah's age three years ago.

Solution

(i) Alishah's present age = x years
Alishah's age before six years = $x - 6$

(ii) $6(x + 5)$

(iii) $\frac{1}{2}(x - 6)$

(iv) Alishah's younger sister's age = $\frac{1}{2}(x - 3) - 5$

6.3.1 Terms of Algebraic Expression

Remember
Operation of multiplication (×) and division (÷) never connect the terms of algebraic expressions.

Different parts of an algebraic expression are connected with each other by an addition (+) or subtraction (−) signs.
Each separated part of the expression is known as term.
For example, $m + 3n + 6$ is an algebraic expression, here m, $3n$ and 6 are the first, second and third terms respectively.

Example 5 Separate each term of the following algebraic expressions.

(i) $a + b$
(ii) $3a + 5b + 9$
(iii) $8x + 4yz - 3$
(iv) $\frac{x}{y} + 4t + 1$

Solution

	Algebraic expression	Terms of algebraic expression	Number of terms
(i)	$a + b$	a, b	2
(ii)	$3a + 5b + 9$	$3a, 5b, 9$	3
(iii)	$8x + 4yz - 3$	$8x, 4yz, -3$	3
(iv)	$\frac{x}{y} + 4t + 1$	$\frac{x}{y}, 4t, 1$	3

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Note
- If any term of an algebraic expression consists of a numeral only, it is known as constant term i.e 9, –3 and 1 in example 5 (ii, iii, iv).
- If any term of an algebraic expression contains variables only, it is known as variable term i.e a, b in example 8 (i).

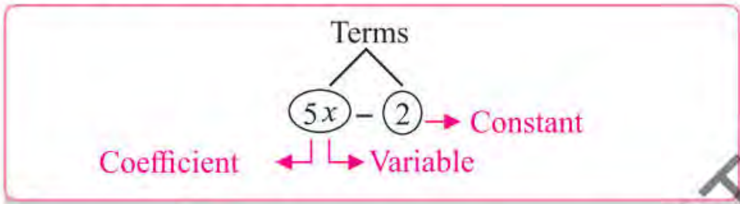
6.3.2 Explain the Terms, Coefficients and Constants in an Algebraic Expression

Coefficients

In algebraic expression we always use a number or symbol to multiply with variable, that number or symbol is called multiplying factor of variable.

"The multiplying factor of any variable in an algebraic expression is known as coefficient".

Look at the below diagram.



Remember
Any variable with no number have a coefficient 1 i.e x is realy 1 x.

Example 6 Write the coefficients of each term of the expressions.

(i) $3a + 4b + c$ **(ii)** $4x + y - 3z$

Note Sometimes in an algebraic expression alphabet letters represent coefficients. i.e. In expression $ax + bx + cx$, x is variable and 'a', 'b' and 'c' are coefficients.

Solution

	Algebraic expression	Terms of algebraic expression	Coefficients
(i)	$3a + 4b + c$	$3a, 4b, c$	3, 4 and 1
(ii)	$4x + y - 3z$	$4x, y, -3z$	4, 1 and –3

Constant terms

Look at the following cases.

Case – I: Consider the algebraic expression

$5a + 2b + 4$. If $a = 5$ and $b = 3$ then $5a + 2b + 4 = 5(5) + 2(3) + 4$
$= 25 + 6 + 4$

Case – II: If $a = 2$ and $b = 4$ then

$5a + 2b + 4 = 5(2) + 2(4) + 4$
$= 10 + 8 + 4$

In both cases we can observe that the third term does not change because 4 is constant. Similarly, in expressions $4x + 5$ and $2p + 4q + 1$, the numbers 5 and 1 are constants. Similarly, in expressions $3x + 2$ and $4p + 5q + 1$, the numbers 2 and 1 being independent of variable are constants.

Thus,

"In an algebraic expression number appearing independent of a variable is called a constant term".

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Example 7 Identify the variable, constant and coefficients of each term in the following algebraic expressions. **(i)** $8x + 9y + 6$ **(ii)** $3a + 4b - 2$ **(iii)** $4xy - 3$

Solution

(i)	(ii)	(iii)
$8x + 9y + 6$	3a + 4b – 2	$4xy - 3$
Variables: x, y	Variables: a, b	Variables: xy
Constant: 6	Constant: –2	Constant: –3
Coefficients: 8, 9	Coefficients: 3, 4	Coefficient: 4

6.3.3 Like and Unlike Terms

Thinking Time

Can you write 2 × 9 as 29? Explain.

Like terms

Look at the following:

Aleezey takes away an order of three packets of samosas from a samosa's shop.

Remember

In an algebraic expression the addition and subtraction can only be performed with like terms.

2 samosas + 4 samosas + 5 samosas = 11 samosas

As, all three packets contain samosas, if she put all packets of samosas in large packet then the large packet will contain 11 samosas.

In algebra Aleezey can write it as: $2s + 4s + 5s = 11s$

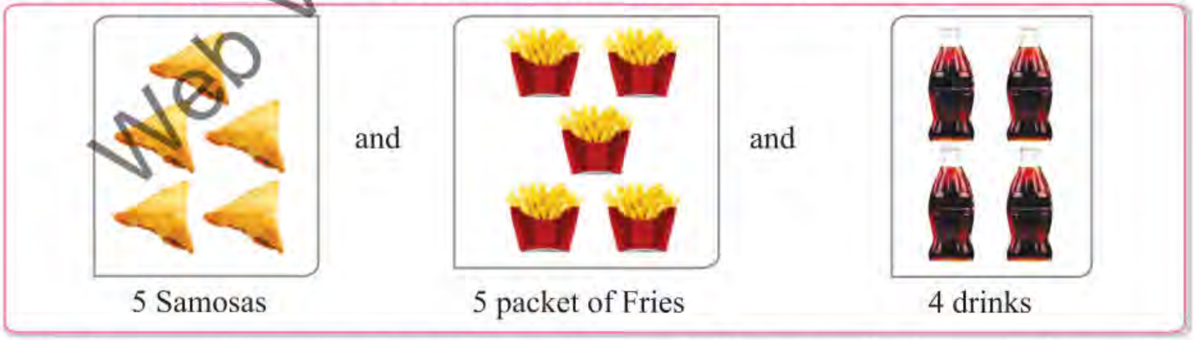
So, *"The terms having the same variables with same indexes are known as like terms."* The like terms can be combined to give a single term.

Note

In the above expression, the power of variable in each term is 1. Each term is different by their variable.

Unlike terms

Shahbaz takes away an order of the following terms from a restaurant.



As all three packets contain different items. If he put all three packets of different items in a large packet then the large packet will contain 5 samosas, 5 packet of fries and 4 drinks.

In algebra Shahbaz can write it as: $5s + 5f + 4d = 5s + 5f + 4d$

So, *"The terms having the different variables are known as unlike terms."*

Unlike terms can never combine to give the single term.

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Remember In an algebraic expression the unlike terms cannot perform addition or subtraction.

Go Online Visit the below link and play online games to identify algebraic terms.
http://www.mathgames.com/skill/6.6-identify-terms-coefficients-and-monomials

Exercise - 6.3

1. Write the following statements in algebraic expressions:

(i) The sum of $3a$ and 8 **(ii)** Difference of $5p$ and 5 **(iii)** Divide $5a$ by $8b$
(iv) Product of a number n and four times a variable y.
(v) Cost of n litres of cooking oil at Rs. 170 per litre.

2. Waseem is two times as old as Umar and Umar is three years older than Amir. If Umar is n years old then write an algebraic expression of:

(i) Waseem's age **(ii)** Umar's age **(iii)** Amir's age

3. Write the algebraic terms of the following expressions:

(i) $n + 3$ **(ii)** $2n + 6m$ **(iii)** $\ell \times m \times n$ **(iv)** $4xy + 6y - 5$ **(v)** $ab + 2$

4. Write the coefficients of each term in the following algebraic expressions:

(i) $4a + 3b$ **(ii)** $23ab$ **(iii)** $\frac{2}{5}xyz$ **(iv)** $pqr + 85$ **(v)** $15p + 81q + 9$

5. Separate the constants, variables and coefficients of each term in the following algebraic expressions:

(i) $8a + 1$ **(ii)** $3a + 2b - 4$ **(iii)** pq **(iv)** $ax + by + c$ **(v)** $3a + 4b + c$

6. Saparate like terms in the following:

$3a$, $21m$, $5a$, $8p$, $4m$, $2s$, $3p$, 8ℓ, $9a$, $76a$, $17bc$, $9cb$.

7. Write the algebraic expressions by adding the following terms:

(i) x, y **(ii)** $a, 2a, 3a$ **(iii)** $x, -y, 3x$ **(iv)** p, pq, q, r

(v) $12a^2, 3b^2$ **(vi)** $\frac{1}{y}, \frac{2}{x}, \frac{3}{2}$ **(vii)** $2xy, yz, 4zx$ **(viii)** $4x, 5x, 6x^2$

8. There are 25 girls and 27 boys in grade 6 in a school, and 30 boys and 23 girls in grade 7 in the same school. Write an algebraic expression to find the sum of girls and boys in both the grades.

9. Arslan is working at a burger shop. On Monday, he sold 40 burgers, 25 fries and 60 cold drinks. On Tuesday, he sold 35 burgers, 28 fries and 54 cold drinks. Write an algebraic expression and find how many burgers, fries and cold drinks did he sell in both the days?

Teaching Point

Take a simple example like $2x + 2y - 9$ to explain the coefficients, variables and constants. Begin by asking students to identify the terms in the expression and then use the expression to draw attention to coefficient, variables and constants.

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Project - 3 Visit your nearest market. Choose any two or three food items and form at least two expressions using the concept of like and unlike terms.

6.4 ADDITION AND SUBTRACTION OF GIVEN ALGEBRAIC EXPRESSIONS

Addition

In the given algebraic expressions only like terms can be added to give a single term.

Look at the following examples for the procedure of addition.

Example 8 Add $4xy$, $3xy$ and $21xy$.

Solution

Vertical method of addition	Horizontal method of addition
$\begin{array}{r} 4xy \\ 3xy \\ +21xy \\ \hline 28xy \end{array}$	$4xy + 3xy + 21xy$ $= (4 + 3 + 21)xy$ $= 28xy$

Remember

- While adding vertically, like terms are arranged in same coloums and then added.
- While adding horizontally like terms are arranged together and added.

Example 9 Find the sum of following algebraic expressions using vertical method of addition.

$(8a + 4b - 2)$, $(8a + 7b - 4)$ and $(23a + 3b - 7)$

Solution

Vertical method of addition.

$$\begin{array}{rrr} 8a & + 4b & - 2 \\ 8a & + 7b & - 4 \\ +23a & + 3b & - 7 \\ \hline 39a & + 14b & - 13 \end{array}$$

Practice -2

(i) Add $3x + 2y$ and $4x - 7y$.

(ii) Find the sum of $3a + 2b - 5c$ and $4a - b + 3c$

Example 10 Find the sum of the following algebraic expressions using horizontal method of addition.

$(ab + 3pq + 5rs)$, $(-4ab - 6pq + 7rs)$ and $(4ab + 2pq)$

Solution Horizontal method of addition

$(ab + 3pq + 5rs)$, $(-4ab - 6pq + 7rs)$ and $(4ab + 2pq)$

$(ab + 3pq + 5rs) + (-4ab - 6pq + 7rs) + (4ab + 2pq)$

Arrange the terms having same variables.

$= ab - 4ab + 4ab + 3pq - 6pq + 2pq + 5rs + 7rs$

$= (1 - 4 + 4)\,ab + (3 - 6 + 2)\,pq + (5 + 7)\,rs$

$= ab - pq + 12rs$

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Subtraction

As we know that subtraction is the reverse process of addition. In the given algebraic expressions the subtraction can be performed by changing the sign of each term of the expression to be subtracted and use the same procedure which we did in addition of algebraic expression. Now look at the following examples.

Example 11 Subtract: $17a$ from $30a$.

Solution

Vertical method of subtraction	Horizontal method of subtraction
$30a$ $\pm 17a$ $13a$	$(30a) - (+17a)$ $= 30a - 17a$ $= (30 - 17)a$ $= 13a$

Example 12 Find the difference of $6a - 5b + c$ and $2a + 2b - 8c$ using vertical method of subtraction.

Solution Vertical method of subtraction.

$$\begin{array}{r} 6a - 5b + c \\ \pm 2a \pm 2b \mp 8c \\ \hline 4a - 7b + 9c \end{array}$$ (by changing the signs)

Remember
- Always add the terms having same signs.
- Always subtract the terms having different signs.

Example 13 Subtract $4xy - 3yz + 4xz$ from $30xy + 5yz - 8xz$ using horizontal method of subtraction.

Solution Horizontal method of subtraction

$(30xy + 5yz - 8xz) - (4xy - 3yz + 4xz)$

$= 30xy + 5yz - 8xz - 4xy + 3yz - 4xz$

Arrange the terms having same variables $= 30xy - 4xy + 5yz + 3yz - 8xz - 4xz$

$= (30 - 4)xy + (5 + 3)yz - (8 + 4)xz$

$= 26xy + 8yz - 12xz$

Practice -3

(i) $5a - 3b$ from $8b - 4c$.

(ii) $17x + 4y$ from $-37x - 13y$.

Exercise - 6.4

1. Add the following terms:

(i) x, x, x, x, x (ii) $3p, 4q, 8p, 9q$ (iii) $4a, 5a, 6a$ (iv) $4xy, 23xy, 3xy$

(v) $98pq, 2qr, 23rs$ (vi) $xy, 14yx, 3pq, 3pq, 54$ (vii) $8x, 2y, 5x, 6y, 3z, 9z, 2y$

2. Sum of the following expressions:

(i) $3p + 4q, 8p - 3q$

(ii) $\frac{5}{7}a + \frac{2}{3}b, \frac{1}{7}a - \frac{1}{3}b$

(iii) $5xy + 9yz, xz - 2yz, 9xy + 4yz$

(iv) $31x + 3y - 4z,\ 6x + 7y - 5z,\ x + y + z$

(v) $9p + 8q + 1, 6q - 3r + 2, 4p + 2r + 7$

(vi) $6p, 2q, 3q + 9r, 4r, -6p$

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3. Subtract:

(i) $3a + 2b$ from $15a + 25b$

(ii) $35xy + 3z$ from $48xy - 4z$

(iii) $16x + 2y - 3z$ from $25x - 3y + z$

(iv) $\frac{1}{3}\ell + \frac{3}{5}m - 8$ from $-\frac{1}{3}\ell + \frac{1}{5}m + 6$

(v) $27\ell + 3m - 4n$ from $-22\ell + 14m + 4n$

4. Solve:

(i) $(4x + 4y) + (5y - 4z) - (5x + 4y + 8z)$

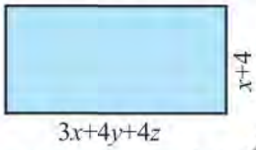
(ii) $(7xy + 3yz) + (8xy - 6yz) - (9xy + 3yz + 2)$

(iii) $(4m + 5m - 2) - (3m + 2n)$

(iv) $(13p + 2q) - (6q - 2r) + 5r$

5. Kinza, Mahak and Rabia have p, $2q$ and $3r$ books respectively. How many books they have altogether?

6. Find perimeter of rectangle?



7. If the perimeter of a triangle is $5x + 6y + 8z$ and the lengths of its two sides are $x + y + z$ and $x - 2y + 3z$ respectively. Find the length of third side of triangle.

Project - 4 Make a booklet for the rules of addition and subtraction of algebraic expression.

6.5 SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS GROUPED WITH BRACKETS

In an algebraic expression, when more than one operations are performed with several terms. Before evaluation of it we need to simplify. To simplify an algebraic expression we will use the same rule which we used in arithmetic expressions. We are familiar that brackets are used to indicate the order for performing the operations. We have learnt four kinds of brackets in unit – 5. These are the four basic grouping symbols or brackets. The operation involving these symbols are performed in the order shown below.

- ——— Vinculum — 1st
- ( ) Parentheses or curved brackets or round brackets. — 2nd
- { } Braces or curly brackets. — 3rd
- [ ] Square brackets. — 4th

We will use the following algebraic expressions by grouping like terms:

Example 14 Simplify the following algebraic expressions by grouping like terms:

$2a + 4b + 5a - 8b - 3a + 8c + 9$

Solution

$2a + 4b + 5a - 8b - 3a + 8c + 9$

$= 2a + 5a - 3a + 4b - 8b + 8c + 9$ (combine the like terms)

$= (2 + 5 - 3)a + (4 - 8)b + 8c + 9$

$= 4a - 4b + 8c + 9$

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Example 15 Simplify the following algebraic expressions:

(i) $5x + 3(8x + 4y)$ **(ii)** $3a + [3b - \{5c + (2b + 4c)\}]$

Solution

(i) $5x + 3(8x + 4y)$
$= 5x + 24x + 12y$
$= 29x + 12y$

(ii) $3a + [3b - \{5c + (2b + 4c)\}]$
$= 3a + [3b - \{5c + 2b + 4c\}]$
$= 3a + [3b - \{2b + 9c\}]$
$= 3a + [3b - 2b - 9c]$
$= 3a + [b - 9c]$
$= 3a + b - 9c$

Note
The sign before the brackets is very important if it is plus (+) then remove the brackets without changing the signs of the each term inside the brackets. If it is minus (–) then change the signs of each term inside the bracket, while removing it.

6.5.1 Evaluation and Simplification of an Algebraic Expression

"The process of finding the value of an algebraic expression by substituting the numeric value in place of variable in each term of the expression, is known as evaluation".

Look the following examples. To evaluate an algebraic expression.

Example 16 If $a = 2$, $b = 4$, $c = 3$, and $d = 5$ then evaluate the following expressions.

(i) $a + bc + 3d$ **(ii)** $4a + 2bc + d - ab$ **(iii)** $2ab + 3cd - (5ba - 4cd)$

Solution

(i) $a + bc + 3d$
Substitute the values $= (2) + (4)(3) + 3(5)$
$= 2 + 12 + 15 = 29$

(ii) $4a + 2bc + d - ab$
Substitute the values $= 4(2) + 2(4)(3) + (5) - (2)(4)$
$= 8 + 24 + 5 - 8 = 29$

(iii) $2ab + 3cd - (5ab - 4cd)$
Substitute the values $= 2ab + 3cd - (5ab - 4cd)$
$= 2ab - 5ab + 3cd + 4cd$
$= -3ab + 7cd$

Now, Substitute the values $= -3(2)(4) + 7(3)(5)$
$= -24 + 105$
$= 81$

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Exercise - 6.5

1. Simplify the following:

(i) $x + y + 3x + 2y + 4x$ **(ii)** $a + a - (3a \div 3)$

(iii) $3a + \frac{1}{3}b + 3\{a + (2b - a + 4b \times 3)\}$ **(iv)** $4x + \{5a + (2a + (3a - 9a \div 3)\}$

(v) $a + [b + \{a - (b + \overline{a - b})\}]$ **(vi)** $[\{(2x + 4y) + 2z\} - 3y] + 2x - 12y$

2. Waqas, Waqar and Saif are selling bags of sweets. Waqas is selling 8 bags of sweets, Waqar is selling 12 bags of sweet and Saif is selling 20 bags of sweets. However Waqar decided to eat three of his bags of sweets as he was hungry. Saif decided to give 2 bags of sweets to his brother. Create an algebraic expression that represents the total amount of bags of sweets that they were selling altogether. Then simplify the expression.

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3. Evaluate the following when $x = 2, y = -5$ and $z = 8$

(i) $4x + y - z$ **(ii)** $2xy + yz - zx$ **(iii)** $4x + \left(\frac{1}{2}y + \frac{1}{14}z\right)$

(iv) $y(3x - z + 2y) + 2$ **(v)** $\frac{x(2y+3z)}{3zx}$ **(vi)** $\frac{x+y}{z} + \frac{y+z}{x}$

4. If $a = 5, b = 8$ and $c = 20$ then evaluate: $b^2 - 4ac$.

5. Express the following into the simplest form and then evaluate it when $x = 8, y = 10$
$3x - 2 + [y + x - \{4 - x + (5 + y)\}]$

Review Exercise 6

1. Each of the questions below is followed by four suggested options. In each case choose the correct option.

(i) 28 + 6 is:
- **a.** a geometric figure
- **b.** an algebraic expression
- **c.** an arithematic expression
- **d.** three dimensional figure

(ii) p + 2q is:
- **a.** a geometric figure
- **b.** an algebraic expression
- **c.** an arithematic expression
- **d.** three dimensional figure

(iii) In algebra a value that can be changed is called:
- **a.** constant **b.** variable **c.** literal **d.** coefficient

(iv) In algebra a fixed value that can never be changed is called: .
- **a.** constant **b.** variable **c.** literal **d.** coefficient

(v) In algebraic expression we can add or subtract only:
- **a.** like terms **b.** unlike terms **c.** literal **d.** constant terms

(vi) The only operations that connect the algebraic terms are:
- **a.** $+, -$ **b.** $\times, \div$ **c.** $+, -, \times$ **d.** $\times$

(vii) To simplify the algebraic expression we use:
- **a.** exponent rule **b.** BODMAS rule **c.** DMAS rule **d.** chain rule

(viii) In expression $4x + 5$,4 is:
- **a.** exponent **b.** variable **c.** constant **d.** coefficient

(ix) Each number in the pattern is called:
- **a.** term **b.** expression **c.** equation **d.** constant

(x) The next term in the sequence 1, 1, 2, 3, 5, 8, ...
- **a.** 10 **b.** 12 **c.** 13 **d.** 15

2. Define the following in a short way:

(i) Variable **(ii)** Constant **(iii)** Coefficient **(iv)** Pattern

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3. Find the sum of:

(i) $2x+3y-4z$ and $5x+16x-9y$ **(ii)** $13xy+2yz, 5yx-2xz$ and $4xy+3zx$

4. Subtract the second expression from the first:

(i) $3x+1, x-y$ **(ii)** $5x-3y-2z, 7x+y-3$

5. Write the coefficients, variables and constants in each term of the following expression:

(i) $ap + bq + c$ **(ii)** $2.5x+\frac{1}{3}y+14$ **(iii)** $13p + 2q + 5$

6. Solve: **(i)** $(13p+2q)+(12p-q)-(8p+3q)$

(ii) $(2xy+3yz+4zx)-(6xy-2yz-5)+(8xy+2yz-35)$

7. Find the perimeter of square field if length of its each side is $3x-y+2z$.

8. Simplify the following expressions:

(i) $(p+q)-(2r-3s)+(p+5s)$ **(ii)** $(x+y)-(5x+\overline{3x-4})$

(iii) $6x-[3x+\{2x-4(\overline{x-2})+y\}]$

9. If $x = 5$ and $y = 7$ then evaluate $6x-[2x+\{2x-4(\overline{x-2})+y\}]$

10. Write the missing numbers in the sequence below.
1, 4, 9, 16, 25, ________, ________, 64, 81. Also, state the rule.

Challenge

Use pattern of four straight lines to join all 9 dots given on the mobile screen.
Do not lift your pen or pencil while performing task.

Summary

- Algebra is a system of symbols and letters of alphabet to replace numbers and find the secret.
- A pattern is some phenomenon that repeats regularly based on a set rule or condition.
- Pattern can be ascending, descending or multiples of a certain numbers.
- Each term in the sequence/pattern is called term.
- A variable is a symbol or letter of alphabet that represents a quantity, the value of which is not known.
- A constant is a fixed value that can never be changed. i.e, all numerals are constants.
- Algebraic expression is an expression built up by the combination of integers and constants, variables are connected with the basic algebraic operational signs (+, −, ×, ÷).
- Each separated part of an algebraic expression is known as term.
- The terms having the same variables and exponents are known as like terms.
- The terms having the different variables and exponents are known as unlike terms.
- The process of finding the values of an algebraic expression by substituting the numeric value in place of variable in each term of expression is known as evaluation.

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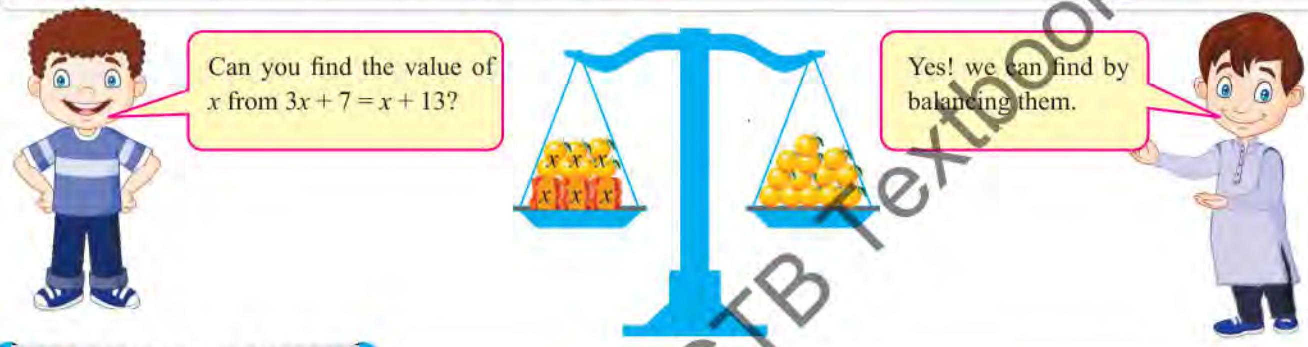
Sub-Domain 7

LINEAR EQUATIONS

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Recognize algebraic equations.
- Differentiate between linear algebraic equations and linear algebraic expressions in one variable.
- Solve linear equations and apply them in real life situations.



INTRODUCTION

An equation means that two things are equal and it is always denoted by an equal (=) sign. It can be compared to the pans of common balance in an equilibrium. The two sides of an equation are same as the two pans of scale and the equality sign shows that the two pans are equal. Take a look on the picture given on the right side where in arithmetics.

$$3 + 1 = 6 - 2$$

The equation says what is on the left side (3 + 1) is equal to what is on the right (6 − 2). The above weighing balance is an excellent example of an equation that we can observe in our daily life, in which.

(i) The two pans of the balance can be considered as two sides of an equation.

(ii) Equal sign "=" indicates that the two pans are in balance.

(iii) When we multiply, divide, add, or subtract on both sides the equation remains in balance.

So, we can say that an equation is like a statement "this is equal to that" so,

"An equation is a statement that shows two mathematical expressions are equal".

7.1 ALGEBRAIC EQUATION

We know that an equation is a statement which shows that two mathematical expressions are equal. Now, consider the following:

Aslam and Neelam write the algebraic sentences as equation. i.e.

Aslam's equation: Six times three increased by seven is twenty five $6(3) + 7 = 25$

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Aslam's equation is a closed sentence, because it contains no variables. A closed sentence is either true or false. To determine whether an equation is true or false:

- Simplify each side of the equation by using BODMAS rule.
- Compare each side of the equation.

$$6(3) + 7 = 25$$
$$18 + 7 = 25$$
$$25 = 25 \text{ true}$$

The equation, 6(3)+7= 25 is a true statement.

Neelam's equation: Twice a number n plus seven equals twenty seven $2n + 7 = 27$

Neelam's equation is an open sentence because it contains a variable n. An open sentence is neither true nor false. To determine wheather a value is a solution of an equation:

- Replace the variables with the given values.
- Simplify using the BODMAS rule and determine the value of variable which makes the statement true.

$2n + 7 = 27$ when $n = 10$	When $n = 11$
$2(10) + 7 = 27$	$2(11) + 7 = 27$
$20 + 7 = 27$	$22 + 7 = 27$
$27 = 27$ true	$29 = 27$ false
$2n + 7 = 27$	

Practice -1

Find the solution of each equation by observation.

(i) $4p - 3 = 5$ (ii) $6x + 1 = 13$

It is the solution of the equation $2n + 7 = 27$ which makes a true statement.

So, Aslam's equation is an arithmetic equation and Neelam's equation is an Algebraic equation. Thus, *"An algebraic equation is an open mathematical sentence which contains an equal sign '='.*

Some other examples of an algebraic equation are $n + 25 = 27$, $3n - 28 = 19$, etc.

Example 1 What is the value of x if

(i) $x + 7 = 10$ (ii) $x - 18 = 2$ (iii) $4x = 24$ (iv) $\frac{x}{3} = 3$

Solution

(i) The value of x is 3 because $3 + 7 = 10$.

(ii) The value of x is 20 because $20 - 18 = 2$.

(iii) The value of x is 6 because $4 \times 6 = 24$.

(iv) The value of x is 9 because $\frac{9}{3} = 3$

Example 2 Write an algebraic expression for the following statements. Use x as variable.

(i) One third of a number increased by sixteen.

(ii) Twice times the cost of a pen decreased by Rs. 7.

(iii) Two added to a number, multiplied by 15.

(iv) Three times a number decreased by 4.

Remember

a, b, c and x, y, z are written in alphabetical order.

Solution (i) $\frac{1}{3}x + 16$ (ii) $2x - 7$ (iii) $2 + 15x$ (iv) $3x - 4$

Thinking Time To multiply a given number by 99. Simply multiply the given number by 100 and subtract the given number from the product.

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7.1.1 Difference between Algebraic Equations and Algebraic Expressions in One Variable

We have learnt in sub-domain 6 about algebraic expressions that an algebraic expression is a mathematical phrase, it is a combination of numbers, variables and mathematical operations. For example,

$$4x,\ 2a+6,\ 3a+2b-7 \text{ and } \frac{1}{2}x+5 \text{ etc.}$$

But an algebraic equation is formed by two algebraic expressions which are connected by the sign of equality '='.

For example, $2a=8$, $3x+5=17$ and $5a+2b-2=2x+4$ etc.

Exercise - 7.1

1. Read and write the following sentences as an arithmetic or algebraic equation:

(i) Four times a number nine increased by four is forty.
(ii) Twice a number four decreased by two is six.
(iii) Twice a number n minus three equals twenty .
(iv) Half of a number x increased by six equals twenty two.
(v) Three times six divided by three is six.
(vi) A number increased by 7 is 14.

2. Write the following equations into words form:

(i) $3(6)-4=14$ **(ii)** $3n+4=10$ **(iii)** $4(6)\div 6=4$ **(iv)** $6x-3=15$

3. Identify the following as an equation or expression:

(i) $4x+2$ **(ii)** $2m+5+3n$ **(iii)** $3p+2=8$ **(iv)** $7s-5$

(v) $26x+2=54$ **(vi)** $\frac{2}{5}p+\frac{1}{3}=q+6$ **(vii)** $3x+2y+7$ **(viii)** $4x+5y=5x-3y+3$

(ix) $4x-1=7$ **(x)** $\frac{3}{4}x-5=4x$ **(xi)** $4x+\frac{1}{3}y+1$

Practice -2 Think and create at leat 5 linear algebraic equations and 5 linear expressions.

7.2 LINEAR EQUATIONS

7.2.1 Linear Equation in One Variable

Linear equation in one variable is

"The equation where the variable has an exponent or power of 1*"*.

Usually '1' is not shown on the variable (it is understood).

For example, $4x+3=0$, $3x=x-3$

Generally we write linear equation as $ax+b=0$

Remember

Exponent
Coefficient → $3\ x^2$
Variable

In an algebraic expression or an equation if a variable showing no power of exponent it means the variable contains exponent or power '1'.

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7.2.2 Construction of Linear Expression and Linear Equation in One Variable

We know that an algebraic expression is a combination of variables, numbers and basic arithmetic operations. An open sentence can be written as algebraic expression or as an equation. Now consider the following examples:

Example 3 Write an algebraic equation for the following statements:

(i) Five less than twice a number is ten.
(ii) A number x decreased by 8 equals 18.
(iii) Divide a number x by thirteen is 10.
(iv) One sixth of a number x added to six is 10.

Solution

(i) $2x - 5 = 10$ **(ii)** $x - 8 = 18$ **(iii)** $\frac{x}{13} = 10$ **(iv)** $\frac{1}{6}x + 6 = 10$

7.2.3 Solution of Linear Equations

In section 7.1 consider the Neelam's equation. Which is $2n + 7 = 27$

The above equation is true for $n = 10$ and false for any other value of n. The number 10 only makes the equation true.

So, *"The value of variable which makes an equation true is known as the solution of linear equation".*

With the balance method we can solve equation by doing the same operations on both sides of the equal sign. As we discussed in section 7.1 that the two sides of an equation are same as the two pans of the weight balance.

Now, look at the following example,

Where we start removing (or adding) things from (or to) the weight balance until we are left with the equal scales.

Example 4 Consider the equation and solve it for x.

$$3x + 7 = x + 13$$

Solution To make the balance and find the value of x. Take a bag x of oranges and separate oranges for the numbers.

Step I On both sides we have separated oranges.

So, we can remove 7 oranges from both pans without loosing the equilibrium of weight balance.

So, from equation $3x + 7 = x + 13$

Subtract 7 from both sides. $3x + 7 - 7 = x + 13 - 7$

$$3x = x + 6$$

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Step II We also have the x bags of oranges, so, we can remove one x bag of oranges from both pans without loosing the equilibrum of the weight balance.

So, subtract x from both sides of the equation $3x = x + 6$

$$3x - x = x + 6 - x$$
$$2x = 6$$

Step III Two orange bags are equal to 6 oranges. It means $6 \div 2 = 3$

So, divide both sides of the equation by 2.

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

So, the example and explanation will help to solve any type of linear equation.

Note To maintain the equilibrium, add, subtract, multiply or divide by the same number to both the weight balance.

Example 5 Solve the following linear equations:

(i) $n + 6 = 8$ (ii) $x - 9 = 35$

Remember
Do, the same operation on both sides of '=' sign in such a way you are left with the unknown variable.

Solution

(i) $n + 6 = 8$
Subtract 6 from both sides of the equation.
$$n + 6 - 6 = 8 - 6$$
$$n = 2$$

Check
$$n + 6 = 8$$
$$(2) + 6 = 8$$
$$8 = 8$$ which is a true sentence.

(ii) $x - 9 = 35$
Add 9 on both sides of the equation.
$$x - 9 + 9 = 35 + 9$$
$$x = 44$$

Check
$$x - 9 = 35$$
$$43 - 9 = 35$$
$$35 = 35$$ which is a true sentence.

Example 6 Solve the following linear equations:

(i) $3a + 7 = 19$ (ii) $7x + 12 = 4x + 27$

Solution

(i) $3a + 7 = 19$
Subtract 7 from both sides of the equation.
$$3a + 7 - 7 = 19 - 7$$
$$3a = 12$$
Divide both sides of the equation by 3, $\frac{3a}{3} = \frac{12}{3}$
$$a = 4$$

Check
$$3a + 7 = 19$$
$$3(4) + 7 = 19$$
$$12 + 7 = 19$$
$$19 = 19$$
Which is a true sentence.

Teaching Point
Bring the original balance in the classroom to perform the math activity as shown above.

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(ii) $7x + 12 = 4x + 27$

Subtract 12 from both sides of the equation.

$$7x + 12 - 12 = 4x + 27 - 12$$
$$7x = 4x + 15$$

Now, subtract $4x$ from both sides of the equation.

$$7x - 4x = 4x + 15 - 4x$$
$$3x = 15$$

Divide both sides by 3

$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

Check

$$7x + 12 = 4x + 27$$
$$7(5) + 12 = 4(5) + 27$$
$$35 + 12 = 20 + 27$$
$$47 = 47$$

Which is a true sentence.

Go Online

Visit the following online game links to practice algebraic equations.

https://www.mathgames.com/skill/3.75-solve-for-the-variable-with-addition-and-subtraction

Example 7 Solve: **(i)** $\frac{1}{4}x + 6 = x - \frac{1}{3}$ **(ii)** $3.5x + 11 = 1.5x + 21$

Solution

(i) $\frac{1}{4}x + 6 = x - \frac{1}{3}$ (L.C.M of denominators = 3×4)

Multiply the equation on both sides by L.C.M = 12

$$12\left(\frac{1}{4}x + 6\right) = 12\left(x - \frac{1}{3}\right)$$
$$3x + 72 = 12x - 4$$

Add 4 on both sides of the equation

$$3x + 72 + 4 = 12x - 4 + 4$$
$$3x + 76 = 12x$$

Subtract $3x$ from both sides

$$3x + 76 - 3x = 12x - 3x$$
$$76 = 9x$$

Divide both sides of equation by 9

$$\frac{76}{9} = \frac{9x}{9}$$
$$x = \frac{76}{9}$$
$$x = 8\frac{4}{9}$$

Check

$$\frac{1}{4}x + 6 = x - \frac{1}{3}$$
$$\frac{1}{4}\left(\frac{76}{9}\right) + 6 = \frac{76}{9} - \frac{1}{3}$$
$$\frac{19}{9} + 6 = \frac{76}{9} - \frac{3}{9}$$
$$\frac{19}{9} + \frac{54}{9} = \frac{76}{9} - \frac{3}{9}$$
$$\frac{73}{9} = \frac{73}{9}$$

Which is true sentence.

(ii) $3.5x + 11 = 1.5x + 21$

Subtract 11 from both sides of the equation.

$$3.5x + 11 - 11 = 1.5x + 21 - 11$$
$$3.5x = 1.5x + 10$$

Subtract $1.5x$ from both sides of the equation.

$$3.5x - 1.5x = 1.5x + 10 - 1.5x$$
$$2x = 10$$

Divide both sides of the equation by 2

$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

Check

$$3.5x + 11 = 1.5x + 21$$
$$3.5(5) + 11 = 1.5(5) + 21$$
$$17.5 + 11 = 7.5 + 21$$
$$28.5 = 28.5$$

Go Online

Visit the following online game links to practice algebraic equations.

https://www.mathgames.com/skill/3.76-solve-for-the-variable-with-muliplication-and-division

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7.1.4 Real Life Problems Involving Linear Equation

In Mathematics real life problems or word problems are representing situations. A real world problem is represented as text rather than in mathematical notation. In our daily life we face many word problems. To solve these problems we must know:

(i) Which information is given in the problem? **(ii)** Which information is required?

Example 8 Arslan and Waqas bought 30 books together . Waqas bought 8 books. How many books did Arslan buy?

Solution Let Arslan bought = x books

Waqas bought = 8 books

So, $x + 8 = 30$

Subtract 8 from both sides of the equation.

$$x + 8 - 8 = 30 - 8$$
$$x = 22$$

Hence, Arslan bought 22 books.

Check

$x + 8 = 30$

$22 + 8 = 30$

$30 = 30$

Remember

Always assume unknown/required value by a variable such as 'x' or n. Or any of the alphabet.

Example 9 What are the three consecutive numbers whose sum is 48?

Solution Let the first number = x

Second number = $x + 1$

Third number = $x + 2$

So, $x + (x + 1) + (x + 2) = 48$

$$x + x + 1 + x + 2 = 48$$
$$3x + 3 = 48$$

Subtract 3 from both sides of the equation.

$$3x + 3 - 3 = 48 - 3$$
$$3x = 45$$

Divide both sides by 3.

$$\frac{3x}{3} = \frac{45}{3}$$
$$x = 15$$

Hence, first number = 15

Second number = $x + 1$
$= 15 + 1$
$= 16$

Third number = $x + 2$
$= 15 + 2$
$= 17$

Remember

To solve any type of word problem simply follow these four steps.

- Read the given statement again and again until you understand the problem.
- Plan to solve.
- Make solution.
- Recheck the problem.

Check

$x + (x + 1) + (x + 2) = 48$

$15 + (15 + 1) + (15 + 2) = 48$

$15 + 16 + 17 = 48$

$48 = 48$

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Example 10 Neelam bought 70 pomegranate for Rs. 820. Some of them are Rs. 10 each and the rest are Rs. 20 each. How many each kind of pomegranate has she bought?

Solution Let the number of pomegranate Neelam bought at Rs. 10 each $= x$

Number of pomegranate she bought at Rs. 20 each $= 70 - x$

Cost of x pomegranate at Rs. 10 each $= 10x$

Cost of $(70 - x)$ pomegranate at Rs. 20 each $= 20 \times (70 - x)$

So, $10x + 20(70 - x) = 820$

$10x + 1400 - 20x = 820$

Subtracting 1400 from both sides of the equation.

$10x + 1400 - 20x - 1400 = 820 - 1400$

$10x - 20x = -580$

$-10x = -580$

Divide the equation both sides by -10.

$$\frac{-10x}{-10} = \frac{-580}{-10}$$

$x = 58$

Hence, Neelam bought 58 pomegranate at Rs. 10 each and 12 pomegranate at Rs. 12 each.

Check

$10x + 20(70 - x) = 820$
$10(58 + 20(70-58) = 820$
$10(58)+20(12) = 820$
$580+240 = 820$
$820 = 820$

Example 11 The present age of Saeed is 5 times as old as his daughter Rabia. After 20 years, the sum of their ages will be 100. How old was Saeed when Rabia was born?

Solution Let the present age of Rabia $= x$ year

The present age of Saeed $= 5x$

Rabia's age after 20 years $= x + 20$

Saeed's age after 20 years $= 5x + 20$

So, $(x + 20) + (5x + 20) = 100$

$x + 20 + 5x + 20 = 100$

$6x + 40 = 100$

Subtract 40 from both sides from the equation.

$6x + 40 - 40 = 100 - 40$

$6x = 60$

Divide the equation by 6

$$\frac{6x}{6} = \frac{60}{6}$$

$x = 10$

Check

$(x + 20) + (5x+20) = 100$
$(10+20)+(5(10)+20) = 100$
$30+(50+20) = 100$
$30+70 = 100$
$100 = 100$

Hence, the present age of his daughter is 10 years and Saeed's age is 50 years.

So, Saeed's age when Rabia was born $= 50 - 10 = 40$ years.

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Activity -2

1. Construct a linear equation in which the solution is the number of textbooks, work books and notebooks in your bag. The equation should comprise at least two operations.
   $2\,(16 - x) = 62$
2. Pass your paper to the student sitting to your right who will solve the equation and verify that answer.

Exercise - 7.2

1. Identify the linear equations:

(i) $4x + 5 = 8$ **(ii)** $12x + 5 = 15 + \frac{1}{x}$ **(iii)** $18x + 7 = 54$ **(iv)** $\frac{x}{2} + 3 = 25$

(v) $3x^2 = 80$ **(vi)** $ax + b = c$ **(vii)** $a\sqrt{x} + 6 = 9$ **(viii)** $\frac{3}{x} + \frac{1}{2} = \frac{2}{9}$

2. Construct the following linear equations:

(i) When a number n is increased by 8 the result is 18.
(ii) When a number y is decreased by 7 the result is 14.
(iii) The sum of two consecutive numbers is 29.
(iv) Four times of a number n is 23 more than twice the number.

3. Solve the following linear equations:

(i) $x + 6 = 18$ **(ii)** $x + 13 = 20$ **(iii)** $5a - 5 = 20$
(iv) $a - 15 = 20$ **(v)** $2x + 5 = 35$ **(vi)** $3n - 8 = 39$

4. The sum of a number x and its double is 12. Find the number.

5. The sum of a number x and its double is 15. Find the number.

6. Five times of a number x is 100. Find the number.

7. Find the length of each side of square of whose perimeter is 36 cm.

8. When Anoral opened a book there are two pages in front of her. The sum of the page numbers is 161. If one page number is 81, what is the other page number?

9. In a cricket match Ahmed Shahzad and Shahid Afridi enhanced the scored of Pakistan cricket team by 96 runs, if Shahid Afridi scored 22 more than Ahmed Shahzad. Find the score of Ahmed Shahzad.

10. Namra is senior to Shumaila by 18 years. After six years, Namra will be twice as old as Shumaila. Find Namra's present age.

11. A number is divided into two parts, such that one part is 40 more than the other. If the two parts are in the ratio 2 : 3, find the number.

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12. A tractor trolly is loaded with sugar canes. The loaded weight of truck is 54,450 kg. If the sugar canes, weight is four times as heavy as the empty tractor trolly. Find the weight of the sugar canes.

Project - 2 Solve the following for x and write the corresponding letter in the space below that are matches your answer.

(i)	$8 + x = 16$	(A)	**(ii)**	$2x - 8 = 6$	(R)	**(iii)**	$x - 10 = 0$	(B)
(iv)	$4 + 3x = 7$	(P)	**(v)**	$2x + 5 = 9$	(C)	**(vi)**	$4x + 4 = 16$	(B)
(vii)	$9 + 2x = 17$	(E)	**(viii)**	$6 + 2x = 24$	(S)	**(ix)**	$3x - 6 = 3$	(B)
(x)	$3x + 5 = 23$	(Y)						

What do witches put on their hair? Find the secret.

9	2	8	7	4

9	1	5	7	6

Review Exercise 7

1. Each of the question below is followed by four suggested options. In each case choose the correct option.

(i) An equation shows that two mathematical expressions are:

a. unequal b. equal
c. less than equal to d. greater than equal to

(ii) In the expression $6x^4$ the number 4 is:

a. constant b. exponent c. coefficient d. base

(iii) The power of variable in a linear equation is always:

a. 1 b. 2 c. 3 d. 4

(iv) The exponent of variable in a linear equation is also known as:

a. degree b. base c. coefficient d. constant

(v) $4x + 5$ is an:

a. algebraic equation b. algebraic expression
c. arithmetic equation d. arithmetic expression

(vi) $ax + b = 0$ is an:

a. algebraic equation b. algebraic expression
c. arithmetic equation d. arithmetic expression

(vii) The value of variable which makes an equation true is known as:

a. Algebra b. arithmetics
c. solution of an equation d. solution of an expression

(viii) Mathematical word problems represent:

a. sentence b. situation c. variable d. constant

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(ix) To write an equation we use the sign:

a. $+$ b. $-$ c. $=$ d. $/$

(x) $x + 9 = 19$ is true for $x =$ _______:

a. 1 b. 10 c. 2 d. 20

2. Solve the following equations:

(i) $3x + 6 = 2x - 9$ (ii) $8x + 4 = 2x + 28$ (iii) $\frac{1}{2}x + 5 = 10$

(iv) $\frac{1}{2}x + 2 = 5 - \frac{1}{6}x$ (v) $3.5x + 2.5 = 1.5x - 4.5$ (vi) $9x + \frac{3}{10} = x - \frac{2}{7}$

3. Solve the following equations:

(i) $2x + 5 = \frac{1}{2}x - 10$ (ii) $4.3p - 2.5 = -8.85p + 10.5$ (iii) $\frac{3}{7}x + \frac{1}{14} = 2 - x$

(iv) $\frac{1}{x} + \frac{2}{x} = 3$ (v) $4(x - 5) - 8(3x - 5) = 4(x + 1) - 2$ (vi) $\frac{7}{9} = \frac{7}{x}$

4. When a number is added to 4 times of itself, the result is 65. Find the number.

5. Abdullah is two more than 3 times as old as Yaseen. If the sum of both of their ages is 60 years. How old is Abdullah?

6. A number is divided into two parts, such that one part is 20 more than the other. If the two parts are in the ratio 3 : 2, find the number.

Summary

- A relationship of equality between two algebraic expressions is known as equation.
- An algebraic equation is an open Mathematical sentence which contains an equal sign, '='
- The value of variable which makes an equation true is known as the solution of an algebraic equation.
- Do the same operation on both sides of '=' sign in such a way that you left with the value of unknown variable.
- In an algebraic expression or equation if a variable showing no power or exponent, it means the variable contains exponent or power '1'.

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Teaching Point

The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

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Sub-Domain 8 SURFACE AREA AND VOLUME

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- State and differentiate between area and perimeter and their units.
- Recognize the formulae to calculate the area and perimeter, surface area, volume of different 2D and 3D shapes.

If each side of the small square is 1*cm*, then what is the volume of this rubik's cube?

INTRODUCTION

In our daily routine we observe many objects, figures and shapes which look like geometrical shapes, such as square, rectangle, and triangle, etc.
These are 2-Dimensional figures.
Similarly, we also, observe and use different types of solid objects, such as ball, table, milk box, match box and fridge etc.

These are all 3-dimensional objects, e.g. look at the matchbox, it has 6-faces, 8-vertices and 12-edges.

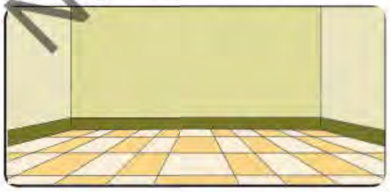
8.1 PERIMETER AND AREA

Perimeter

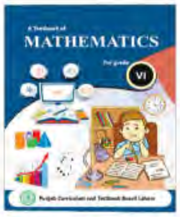
"The distance around any 2-dimensional closed shape is called perimeter."

a.

The length of fence around the field.

b. 

The length of floor of your room.

c. 

The total length of all sides of front page of your math book.

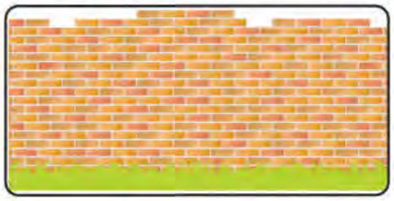
Mostly the distance and length is measured in kilometre (*km*), metre (*m*) and millimetre (*mm*).

Area

The region enclosed within a boundary of a closed shape is called area.

a.

The cost of weeding in the plot.

b. 

The number of tiles in the wall.

c.

The cost of painting the wall.

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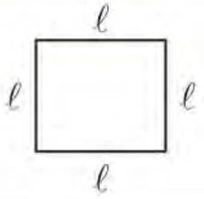
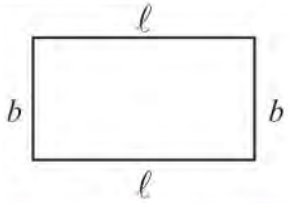
8.1.1 Recognization of Formulae to Calculate the Area and Perimeter

The following are the some important results we covered in our primary classes to calculate the perimeter and area of square and rectangle.

Name	Figure	Perimeter	Area
Square		P = Sum of all sides $= \ell + \ell + \ell + \ell$ $= 4\ell$	$A = \ell \times \ell$ $= \ell^2$
Rectangle		P = Sum of all sides $= \ell + b + \ell + b$ $= 2\ell + 2b$ $= 2(\ell + b)$	$A = \ell \times b$

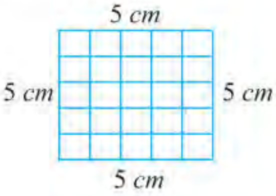
8.1.2 Calculation for Perimeter and Area of a Square and Rectangle

Perimeter and area of square and rectangle can be found by using the formula given in previous section 8.1.1.

Perimeter and area of square

Example 1 Find the perimeter of square whose length of each side is 5 *cm*.

Solution Length of each side of square = $\ell = 5\ cm$



Formula: Perimeter of square = $P = 4\ell$

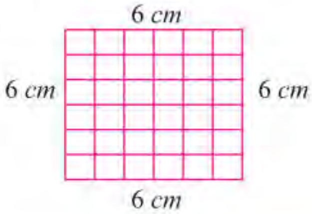
$= 4 \times 5$

$= 20\ cm$

Hence, the perimeter of above square is 20 *cm*.

Example 2 Find the area of square if the length of each side of square is 6 *cm*.

Solution The length of each side of square = 6 *cm*.



Formula: Area of square = $A = \ell^2$

$= (6)^2$

$= 6 \times 6$

$= 36\ cm^2$

Hence, the area of square is 36 cm^2

Teaching Point Ask students to bring a grid sheet or you provide them. Then ask the students to draw a at one rectangular shaped object and one square shaped object on the grid. Finally ask them to calculate area and perimeter of drawn shapes.

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Example 3 The length of each side of square flower bed is 8m. The cost of flowering is Rs. 30 per square metre and the cost of fencing is Rs. 20 per metre. Find the total cost of flowering and fencing the flower bed.

Solution

Length of each side of flower bed $= \ell = 8\ m$

Cost of flowering = 30 per square metre.

Cost of fencing = 20 per metre

Now, Perimeter of square $= P = 4\ell$

Perimeter of flower bed $= 4(8)$

$= 32\ m$

So, the total length of fence is 32 m.

Cost of fencing $= 20 \times 32$

$=$ Rs. 640

As, Area of square $= A = \ell^2$

$= (8)^2$

$= 8 \times 8$

$= 64\ m^2$

Total cost of flowering $= 30 \times 64$

$=$ Rs. 1920

Hence, the total cost of flowering is Rs. 1920 and the total cost of fencing is Rs. 640

Perimeter and area of rectangle

Example 4 Find the perimeter of the rectangle having length of 6 cm and breadth of 4 cm.

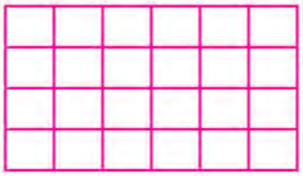
Solution Length of rectangle $= \ell = 6\ cm$, Breadth of rectangle $= b = 4\ cm$

Formula: Perimeter of rectangle $= P = (\ell + b)$

$= 2(6 + 4)$

$= 2(10)$

$= 20\ cm$



Hence, the perimeter of rectangle is 20 cm.

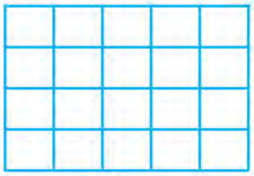
Example 5 Find the area of the rectangle having length of 5 cm and breadth of 4 cm.

Solution Length of rectangle $= \ell = 5\ cm$, Breadth of rectangle $= \ell = 4\ cm$

Formula: Area of rectangle = A = Length × Breadth $= \ell \times b$

$= 5 \times 4$

$= 20\ cm^2$



Hence, the area of rectangle is $20\ cm^2$.

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Example 6 Neelam made a Pakistani national flag for the national day. The flag is $2m$ wide and $3m$ long. The cost of cloth used in flag is Rs. 75 per m^2 and that of the lace used around the flag is Rs. 10 per metre. How much money did she spend to make the flag?

Solution The length of flag $= \ell = 3\ m$

Breadth of flag $= b = 2\ m$

Perimeter of flag = P = 2 (Length + Breadth) $= 2(\ell + b)$

$= 2(3 + 2)$

$= 2(5)$

$= 10\ m$

So, Neelam used $10\ m$ of lace around the flag.

Cost of lace for 1 metre = 10

Cost of lace for 10 metre $= 10 \times 10$

= Rs. 100

Now, area of flag = A = Length × Breadth $= \ell \times b$

$= 3 \times 2$

$= 6\ m^2$

So, Neelam used $6\ m^2$ of cloth to make the flag.

Cost of cloth for $1\ m^2$ = Rs. 75

Cost of cloth for $6\ m^2$ $= 75 \times 6$

= Rs. 450

Total cost of the flag = 100 + 450

= Rs. 550

Hence, Neelam spent Rs. 550 to make a flag for national day.

Practice -1

Find the area and perimeter of the rectangles if

(i) length = $7cm$, breadth = $5cm$

(ii) length = $15cm$, breadth = $13m$

Activity -2

Use scale/ruler in your bag to measure length and breadth of the following objects then find the area and perimeter.

(i) White board/black board

(ii) Mathematics book

(iii) Paper pad

Example 7 Find the breadth of a rectangular field with length $20\ m$ and area $315\ m^2$. Also find the perimeter of the rectangular field.

Solution Area of rectangular field $= 315\ m^2$

Length of rectangular field $= 20\ m$

Area of rectangle = length × breadth

$$\frac{\text{Area of rectangle}}{\text{Length}} = \text{Breadth}$$

So, breadth of rectangular field

$$= \frac{\text{Area of rectangular field}}{\text{Length of rectangular field}}$$

$$= \frac{315}{20} = 15.75\ m$$

Now, Perimeter of rectangle

= 2 (length + breadth)

$= 2(\ell + b)$

$= 2(20 + 15.75)$

$= 2(35.75)$

$= 71.50\ m$

Hence, breadth of rectangular field is $15.75\ m$ and perimeter is $71.50\ m$.

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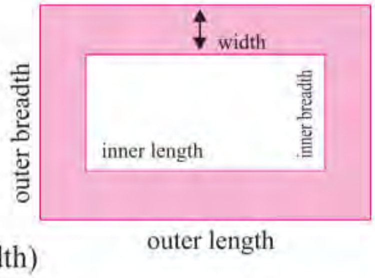
Activity -3 Count the tiles of your classroom's wall. Find the area and perimeter of the classroom wall.

Go Online Use the following online game links to practice area and perimeter.
https://www.splashlearn.com/s/math-games/find-the-perimeter-of-the-shapes-using-girds
https://www.splashlearn.com/s/math-games/find-the-perimeter-of-polygons
https://www.splashlearn.com/s/math-games/find-the-area-by-multiplying-the-side-lengths

8.1.3 Find Area of Path (Inside or Outside) of a Rectangle or Square

We have learnt about perimeter and area of square and rectangle. Now we will discuss about the area of path (inside or outside). It is observed that in a rectangular or square park, garde field, etc. Some space in the form of paths is left inside or outside. We will use the concept of the area of square and rectangle to determine the area of different paths.



Now look at the figure of rectangle.

The coloured portion represents the area of path.

Area of large rectangle = Outer length × Outer breadth.

Area of small rectangle = Inner length × Inner breadth

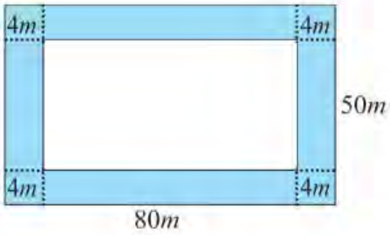
So, Area of path = Area of large rectangle − Area of small rectangle.

= (outer length × outer breadth) − (inner length × inner breadth)

Example 8 A rectangular park of length 80 m and breadth 50 m is to be surrounded externally by a path which is 4 m wide. Find the cost of turfing the path at the rate of Rs. 200 per m^2.

Solution Length of park with path = 80 m

Breadth of park with path = 50 m



Area of park with path = A_1 = Outer length × Outer breadth

$= 80 \times 50 = 4000\ m^2$

Inner length $= 80 - (2 \times 4)$

$= 80 - 8 = 72\ m$

Inner breadth $= 50 - (2 \times 4)$

$= 50 - 8 = 42\ m$

Area of park without path = A_2 = Inner length × Inner breadth

$= 72 \times 42$

$= 3024\ m^2$

Now, area of path of the park $= A_1 - A_2$

$= 4000 - 3024 = 976\ m^2$

Cost of turfing on 1 m^2 = Rs. 200

Cost of turfing on 976 m^2 = 200 × 976 = Rs. 195, 200

Hence, the cost of turfing the park path is Rs. 195 , 200.

Activity -4

Find the area of aluminium or wooden strip of white/black board in your classroom.

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Remember If the path lies all around the park on the outside, then the length of the park including the park = length of the park + 2 × width of the path.
Similarly, the breadth of the park including the path = breadth of the park + 2 × width of the path.

Exercise - 8.1

1. Find the perimeter of the following shapes:

2. Find the area and perimeter of the following shapes:

3. A rectangular certificate is 250 *mm* wide and 180 *mm* long. Find its area and perimeter.

4. Find the length of each side of square painting if its perimeter is 49 *m*.

5. The area of rectangle is 54 cm^2. If the breadth of rectangle is 6 *cm*, find its perimeter.

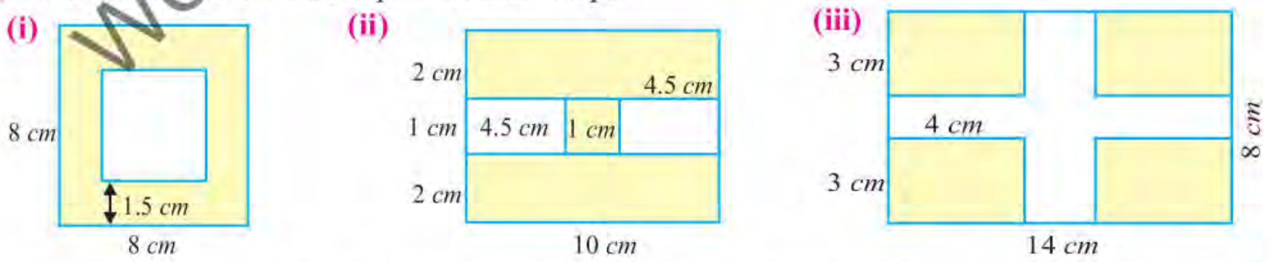
6. Find the breadth of rectangle if its area is 60 cm^2 and length is 10 *cm*. Also find its perimeter.

7. The perimeter of a square garden is 24 *km*. Find its area.

8. Zahid wants to grow plants in a square field. He plants 12 trees in a row. The cost of planting a tree is Rs. 100. How much money will be spent to plant the trees in the field?

9. The length of rectangular field is 60 *m* and the breadth is 50 *m*. A girl wants to fence the field. The cost of fencing the field is Rs. 108 per metre. Find the total cost of fencing the field.

10. Find the area of the shaded parts of each shape:



11. A garden 100 *m* long and 50 *m* wide has a road 3 *m* wide all around the garden. Find the cost of paving the road at the rate of Rs. 250 per m^2.

12. In a square garden having each side of 25 *m*, two paths 6 *m* wide cross each other at right angle at the middle. Find the total area of the paths.

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Project - 1 Calculate the area and perimeter for the cap of your lunch box. Make a layout of your school on a grid sheet and find area and perimeter of your classroom.

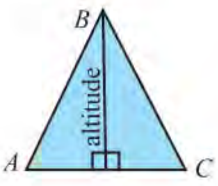
8.1.4 Altitude of Geometrical Figure

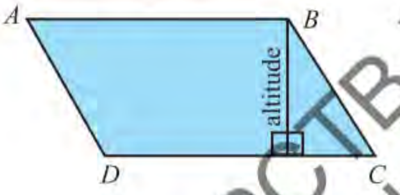
The word altitude means the height of an object above a reference level or sea level i.e.

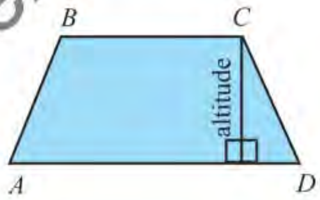
When we are in an aeroplane, the attendant probabily updates us that how high up the aeroplane is in the air. This is known as altitude of the aeroplane. This altitude refers to how high the aeroplane is with respect to sea level. In geometry

'The altitude is measured of the shortest distance between the base and its top"

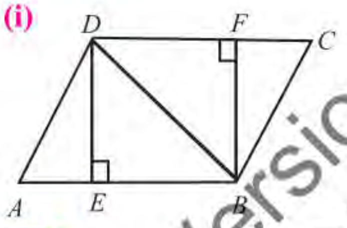
Now, Look at the following geometrical figures.

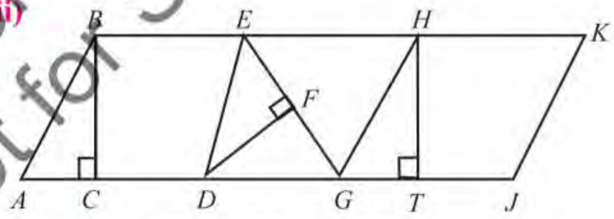






Example 9 Identify the altitudes of the following geometrical shapes:



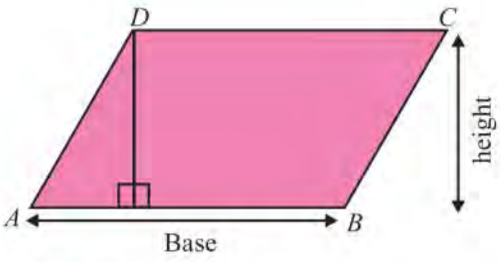


Solution (i) $m\overline{DE}$ and $m\overline{BF}$ (ii) $m\overline{BC}$, $m\overline{DE}$ and $m\overline{HT}$

8.1.5 Area of Parallelogram

Parallelogram

A parallelogram is a 4-sided plane figure formed by two pairs of parallel lines. Opposite sides are equal in length and opposite angles are equal in measurement. The height is fixed. In a parallelogram when we draw perpendicular from any point of its opposite side, then this perpendicular is known as height and opposite side is known as its base.



Remember The altitude always makes the right angle with the base.

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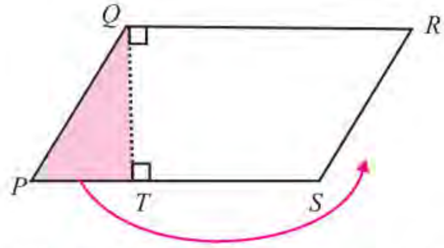
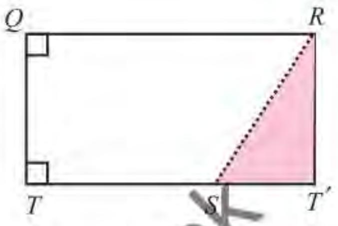
Area of a parallelogram

To find the area of parallelogram consider a parallelogram *PQRS*



A perpendicular $\overline{QT}$ is drawn on the base $\overline{PS}$.
So, a triangle *PQT* formed.



The triangular area is being cut from $\overline{QT}$ and placed with $\overline{RS}$.
So, we get a rectangle *TQRT′*

The above activity shows that a parallelogram can be converted into rectangle. So, the area of rectangle and area of parallelogram is same and we can use the same formula.

Area of rectangle = Area of parallelogram

Length × breadth = $m\overline{PS} \times m\overline{QT}$

Here, the line segment $m\overline{QT}$ is *"altitude"* of the parallelogram *PQRS*, and $m\overline{PS}$ is base.

So, the area of parallelogram = base × altitude.

If we represent base of parallelogram with "*b*" and altitude with "*a*". Then $A = b \times a$

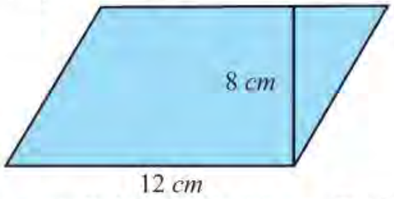
Example 10 Find the area of a parallelogram if its base is 12 *cm* and perpendicular height is 8 *cm*.

Solution Base of parallelogram = $b = 12\ cm$, Perpendicular height of parallelogram = $h = 8\ cm$

Formula: Area of parallelogram = base × perpendicular height

$$A = b \times h$$
$$= 12 \times 8 = 96\ cm^2$$

Hence, the area of parallelogram is 96 cm^2.



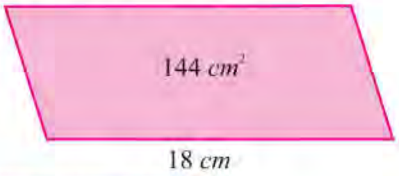
Example 11 The area of parallelogram is 144 cm^2, and the base of parallelogram is 18 *cm*. Find the height of parallelogram.

Solution Base of parallelogram = $b = 18\ cm$, Area of parallelogram = $A = 144\ cm^2$

Formula: Area of a parallelogram = base × perpendicular height

$$A = b \times h$$
$$h = \frac{A}{b}$$
$$h = \frac{144}{18}$$
$$h = 8\ cm$$

Hence, the height of parallelogram is 8 *cm*.



Practice -2

Calculate the area of parallelogram if its length is 13*cm* and altitude is 9*cm*.

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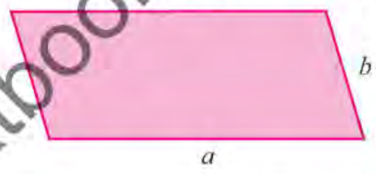
Remember Altitude of a parallelogram is also known as perpendicular height of a parallelogram. So, we can write the formula for the area of a parallelogram as: Area of a parallelogram = base length × perpendicular height

Perimeter of parallelogram

"The total length of the boundaries or all sides of a parallelogram is called its perimeter."

The perimeter of parallelogram is same as the perimeter of rectangle as both shapes have the same properties. Let "a" and "b" are the length of sides of the parallelogram.

We know that the opposite sides of parallelogram are equal. So, we can calculate the parimeter of parallelogram by using the formula.



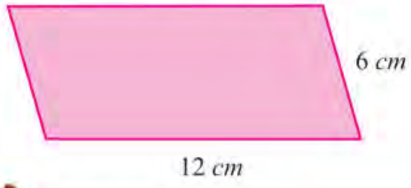
Perimeter of parallelogram, $P = (a + b + a + b)$

$P = 2a + 2b$

$P = 2(a + b)$

Example 12 Find the perimeter of the parallelogram, whose dimensions are 12 cm and 6 cm.

Solution



Base of parallelogram = 12 cm

Side length of parallelogram = 6 cm

Perimeter of parallelogram = $P = 2(a + b)$

$P = 2(12 + 6)$

$= 2(18) = 36\ cm$

Hence, the required perimeter of parallelogram is 36cm.

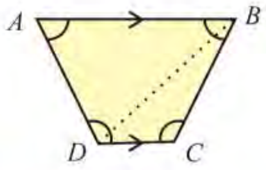
Practice -3

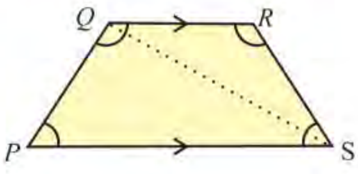
Calculate the perimeter of the parallelogram, whose dimensions are 11 m and 7 m respectively.

8.1.6 Area of a Trapezium

Trapezium

A trapezium is a quadrilateral with four sides and four angles. It has only two parallel sides. The trapezium is also known as trapezoid. i.e.

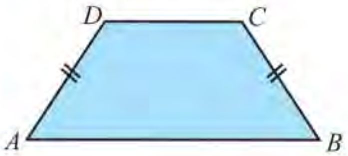




There are three kinds of trapezium.

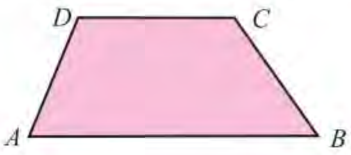
a Isosceles trapezium

If the non-parallel sides of a trapezium are equal in length then the trapezium is called isosceles trapezium.



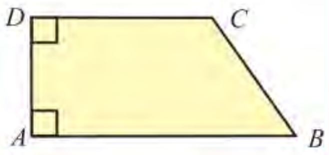
b Scalene trapezium

If all the sides and all the angles of a trapezium are of different measurement then the trapezium is called scalene trapezium.



c Right trapezium

If a trapezium has two right angles then the trapezium is called right trapezium.



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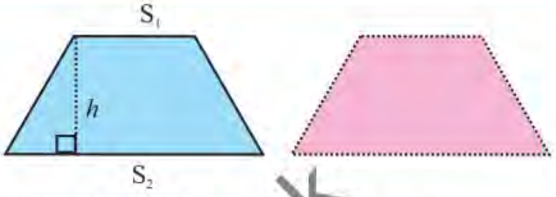
Remember **Regular Trapezium:**

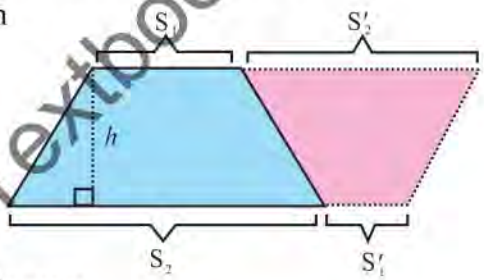
A trapezium whose non-parallel sides are equal in length and its base angles are equal is known as regular trapezium.

Area of trapezium



To find the area of trapezium, we consider the following two congruent trapeziums with perpendicular height h and the opposite sides S_1 and S_2 respectively.

Now, put the above two trapeziums together, as shown in the figure below to form a parallelogram. (We can form a parallelogram by rotating the original trapezium at 180°)



As, Area of parallelogram = base × perpendicular height

$= b \times h$

$= (S_1' + S_2) \times$ perpendicular height $\quad \because S_1 = S_1'$

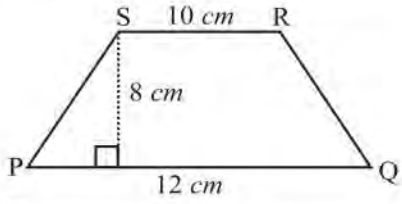
$= (S_1 + S_2) \times$ perpendicular height

The area of original trapezium is one half of the area of the parallelogram.

So, the area of trapezium $= \frac{1}{2}(\text{side}_1 + \text{side}_2) \times$ perpendicular height

$$A = \frac{1}{2}(S_1 + S_2) \times h$$

Example 13 Find the area of trapezium whose length of parallel sides is 10 *cm* and 12 *cm* respectively, with perpendicular height 8 *cm*.



Solution

Longer base of trapezium $= S_1 = 12\ cm$

Shorter base of trapezium $= S_2 = 10\ cm$

Height of trapezium $= h = 8\ cm$

Formula: Area of trapezium $= \frac{1}{2}(\text{side}_1 + \text{side}_2) \times$ perpendicular height

$= \frac{1}{2}(S_1 + S_2) \times h$

$= \frac{1}{2}(12 + 10) \times 8cm$

$= 88\ cm^2$

Hence, the area of trapezium is $88\ cm^2$

Thinking Time

Derive the formula for area of trapezium using area of triangle. You may get help from your teacher.

Practice -4

Find the area of trapezium whose length of parallel sides is 15 *cm* and 12 *cm* respectively, its perpendicular height is 6 *cm*.

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Perimeter of trapezium

"The sum of all sides of a trapezium is called its perimeter." Perimeter can be calculated by using the formula:

Perimeter of trapezium = $P = a + b + c + d$

Where a, b, c and d are the length of sides of the trapezium.

Example 14 If the lengths of parallel sides of a trapezium are 15 *cm* and 10 *cm*, and non-parallel sides are 6 *cm* each. Find its perimeter.

Solution Lengths of parallel sides = 15 *cm* and 10 *cm*

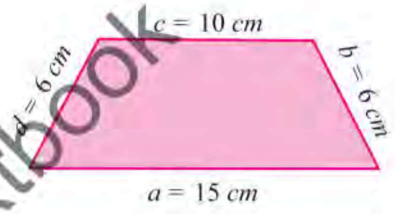
Lengths of non-parallel sides = 6 *cm* each

Perimeter of trapezium = $P = a + b + c + d$

$P = 15 + 6 + 10 + 6$

$= 21 + 16 = 37$ *cm*

Hence, the perimeter of trapezium is 37 *cm*.



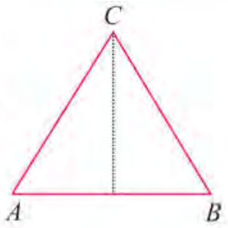
Practice -5

What is the perimeter of trapezium *ABCD*? If $m\overline{AB}$ = 20 *cm*, $m\overline{BC}$ = 4 *cm*, $m\overline{CD}$ = 13 *cm* and $m\overline{AD}$ = 5 *cm*

8.1.7 Area of Triangle

Triangle

We know about triangle that it is a polygon having three sides and three angles. In previous classes we have learnt about different types of triangle such as, equilateral, isosceles, scalene and right angled triangle. In the figure ABC is a triangle.



Area of a triangle

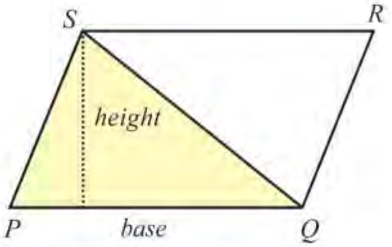
Consider a parallelogram *PQRS* which is divided into two equal triangles. *PQS* and *QRS*.

As, Area of parallelogram = base × perpendicular height

So, The area of the ΔPQS is half the area of parallelogram.

Similarly,

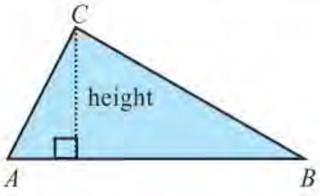
The area of the ΔQRS is also half the area of parallelogram.

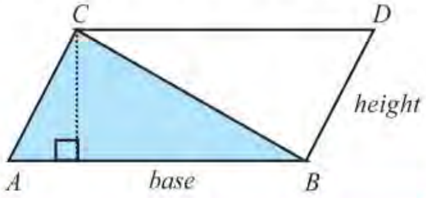


$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$A = \frac{1}{2}(b \times h),$$ This formula can also be used for other triangles.

When we draw opposite parallel sides of triangle *ABC*. We see the area of triangle *ABC* still half the area of the parallelogram *ABCD*.





Remember The height of the triangle is always measured at right angle to the base.

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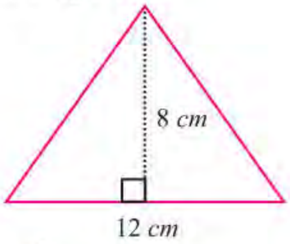
Example 15 Find the area of the triangle having base length 12 *cm* and the height 8 *cm*.

Solution Base length of triangle $= b = 12\ cm$

Perpendicular height of triangle $= h = 8\ cm$



Formula: Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times (b \times h)$$

$$= \frac{1}{2}\ (12 \times 8)$$

$$= \frac{96}{2} = 48\ cm^2$$

Hence, the area of triangle is $48\ cm^2$

Practice - 6

Find the area of triangle *ABC* having base length 9 *cm* and its perpendicular height 5 *cm*.

Remember

Observe the diagram carefully to solve the problem.

Perimeter of a triangle

"The sum of all sides of a triangle is called its perimeter."

To calculate the perimeter of any triangle we use formula

$$\text{Perimeter} = P = a + b + c$$

Where *a*, *b*, and *c* are the length of all sides of given triangle.

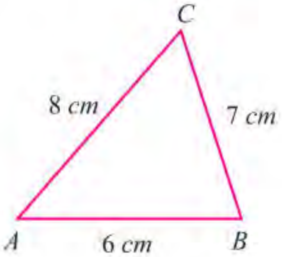
Example 16 Find the perimeter of triangle *ABC* having lengths of sides 6 *cm*, 7 *cm* and 8 *cm* respectively.

Solution

Length of $m\overline{AB} = 6\ cm$

Length of $m\overline{BC} = 7\ cm$

Length of $m\overline{AC} = 8\ cm$



Perimeter of triangle $= P = m\overline{AB} + m\overline{BC} + m\overline{AC}$

$$= 6 + 7 + 8$$

$$= 21\ cm$$

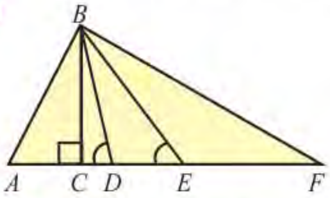
Hence, the required perimeter of the triangle is 21 *cm*.

Practice - 7 Find the length of missing side of triangle whose perimeter is 36 *cm* and two sides are 10 *cm* and 11 *cm* respectively.

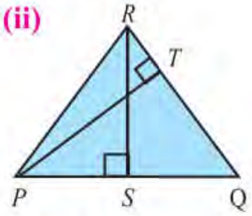
Exercise - 8.2

1. Identify the altitudes of the following figures:

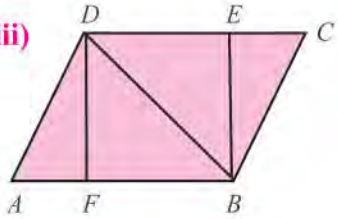
(i)



(ii)



(iii)



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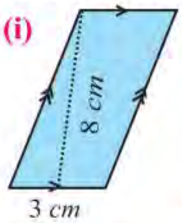
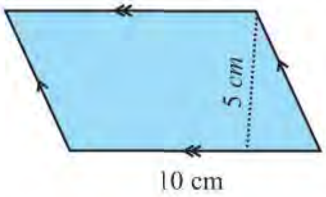
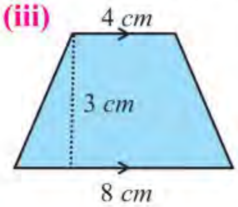
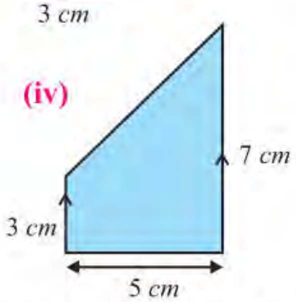
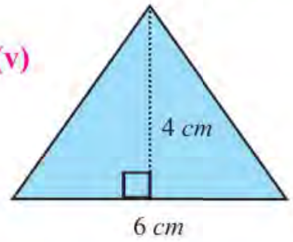
2. Calculate the area and perimeter for each of these parallelograms, trapeziums and triangles:

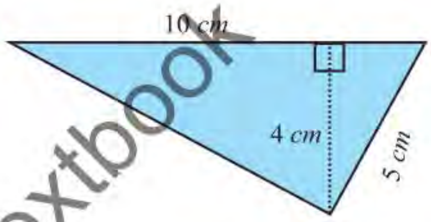
(i)


(ii)


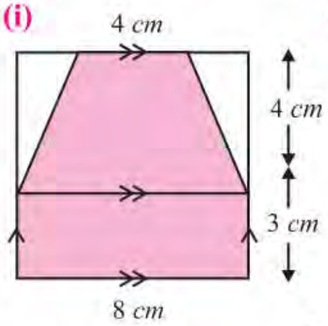
(iii)


(iv)


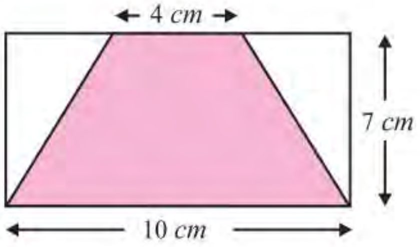
(v)


(vi)


3. Find the area of each shaded shape:

(i)


(ii)

(iii)


4. The area of a parallelogram is 126 cm^2 and its perpendicular height is 9 *cm*. Find its base. Also, calculate its perimeter.

5. The base and area of parallelogram is 9 *cm* and 99 cm^2 respectively. Find its perpendicular height.

6. The parallel sides of a trapezium are 25 *cm* and 30 *cm* and it s area is 192.5 cm^2. What is the height of trapezium?

7. A trapezium has a longer base that is 25 *cm* and the height is 5 *cm*. The area of trapezium is 200 cm^2. What is the shorter base of the trapezium?

8. The height of a triangular garden is 55 *m* and base is 90 *m*. Find the number of flowers in the garden if 20 flowers are in the area of each square metre.

Project - 2 Make a booklet for all the formulae and apply them to self created questions. Create at least one question for area and perimeter of

(i) square **(ii)** rectangle **(iii)** parallelogram **(iv)** triangle.

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8.2 3-DIMENSIONAL SHAPES/FIGURES (CUBE AND CUBOID)

The solid figures are 3 – dimensional figures. These figures have height, width and depth. All solid figures have thickness and can be held in our hand. For example, cube, cuboid etc.

8.2.1 Units of Surface Area and Volume

Surface area

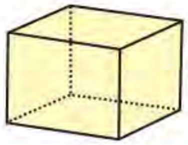
We have learnt about area and perimeter in our previous section. The area of any 2-dimensional figure or shape is the measurement of space occupied by the figure, such as a unit square has the area of 1 square unit.

Similarly, the surface area of any three -dimensional solid figure is the sum of the areas of all of its faces. Which is also measured in square units. For example a cube has 6-faces. If the area of each face is 1 cm^2 then the surface area of cube will be the sum of area of all its faces i.e.

$$1\ cm^2 + 1\ cm^2 + 1\ cm^2 + 1\ cm^2 + 1\ cm^2 + 1\ cm^2 = 6\ cm^2$$

Volume

"The volume is the quantity of three dimensional space enclosed by a closed surface".
The volume of the solid figure like cube or cuboid is the number of cubic units that it contains. The cubic units may be cubic centimetres or cubic metres and so on. If one side length of cube is 1cm then its volume will be $1\ cm \times 1\ cm \times 1\ cm = 1\ cm^3$.



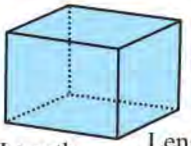
Remember Most of the time the dimensions of cube or cuboid are given in different units. They will be converted into the same unit before calculating the volume and surface area.

8.2.2 Find Surface Area and Volume of Cube and Cuboid

Surface area of cube

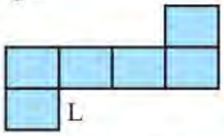
In the cube, the length, breadth and height are same.

$\therefore$ length = breadth = height

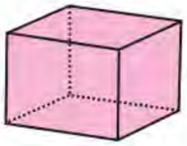


The surface area of cube can be calculated by using the formula.

Formula: Surface area of cube = 6 × (length × length)
= 6 × (length)^2
= $6\ell^2$



Example 17 The following figure shows a cube with 8 *cm* long each side. Find the total surface area of the figure.



Solution Length of each side of cube = ℓ = 8 *cm*

Surface area of cube = $6\ell^2$
= $6(8)^2$
= 6×64
= $384\ cm^2$

Hence, the required surface area of cube is $384\ cm^2$.

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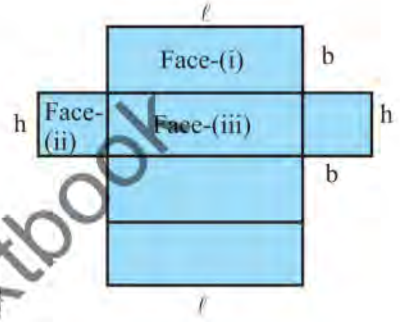
Practice -8 Use formula to find the surface area of cube having length of each side 10cm.

Surface area of cuboid

We are familiar that in cuboid shaped solids, the area of opposite surfaces is same.



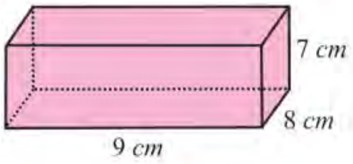
Area of face (i) = length × breadth
$= \ell \times b$
Area of face (ii) = breadth × height
$= b \times h$
Area of face (iii) = height × length
$= h \times \ell$

So, the total surface area of cuboid

= 2 × { Area of face (i) + Area of face (ii) + Area of face (iii)}

$$S = 2\{(\ell \times b) + (b \times h) + (h \times \ell)\}$$

Example 18 The following figure shows a rectangular cuboid 9 *cm* long 8 *cm* wide and 7 *cm* high. Find the total surface area of cuboid.



Solution
Length of cuboid = ℓ = 9 *cm*
Breadth of cuboid = b = 8 *cm*
Height of cuboid = h = 7*cm*
Surface area of cuboid $= 2\{(\ell \times b) + (b \times h) + (h \times \ell)\}$
$= 2\{(9 \times 8) + (8 \times 7) + (7 \times 9)\}$
$= 2[72 + 56 + 63]$
$= 2[191]$
$= 382\ cm^2$

Hence, the total surface area of cuboid is $382\ cm^2$.

Activity-3 Use formula to find the surface area of your new shoes box.

Volume of cube

As we are familiar that the length, breadth and height are same in any cube.

∴ length = breadth = height

So, we can use the formula to find volume of cube.

Volume of cube = length × length × length

$$V = \ell \times \ell \times \ell = \ell^3$$

Practice -9
Find the volume of cube having length of each side 7*cm*.

Teaching Point Ask students one day before to bring empty boxes of cube or cuboid shape in the class, then next day find the volume of those boxes.

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Example 19 A room is shaped like a cube. Its measurement is 9 m on each edge. What is the total volume and total surface area of 5 such rooms ?

Solution Length of an edge of a room = $\ell = 7\ m$.

(i) Surface area

$$\text{Surface area of cube} = 6 \times (\text{length})^2 = 6\ell^2$$
$$= 6(9)^2$$
$$= 6 \times 81$$
$$= 486$$
$$\text{Surface area of 5 rooms} = 5 \times 486$$
$$= 2430\ m^2$$

(ii) Volume

$$\text{Volume of cube} = (\text{length})^3$$
$$V = \ell^3$$
$$= (9)^3$$
$$= 729$$
$$\text{Volume of 5 rooms} = 5 \times 729$$
$$= 3645$$

Hence, the surface area of 5 rooms is 2430 m^2 and the volume of 5 rooms is 3645 cm^3.

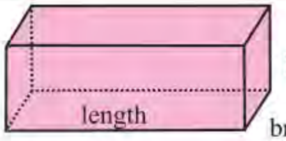
Thinking Time

What happens to the volume of the cube if you double the length of its one edge.

Challenge

Calculate the volume of your sleeping room. You may get help from your parents or siblings.

Volume of cuboid

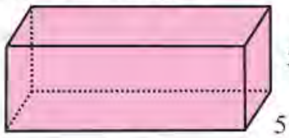


In the cuboid shape solids the measurement of atleast one edge is different from other two edges.

So, we can use the formula to find volume of cuboid = length × breadth × height

$$V = \ell \times b \times h$$

Example 20 Find the volume of cuboid if it is 7cm long, 5 cm wide and 5 cm high.



Solution
Length of cuboid $= \ell = 7\ cm$
Breadth of cuboid $= b = 5\ cm$
Height of cuboid $= h = 5\ cm$

$$\text{Volume of cuboid} = \text{length} \times \text{breadth} \times \text{height}$$
$$V = \ell \times b \times h$$
$$= 7 \times 5 \times 5$$
$$= 175$$

Hence, the volume of cube is 175 cm^3.

Practice -9

A container is 20 m long, 3 m wide and 5 m high. What is the surface area and volume of the container.

Example 21 Find the total surface area of a solid cuboid of volume 560 cm^3 with 10 cm of length and 7cm of breadth.

Solution

Volume of solid cuboid $= V = 560\ cm^3$
Length of solid cuboid $= \ell = 10\ cm$
Breadth of solid cuboid $= b = 7\ cm$

$$\text{Height of solid cuboid} = \frac{\text{Volume of solid cuboid}}{\text{length} \times \text{breadth}}$$

$$h = \frac{V}{\ell \times b} = \frac{560\ cm^3}{10\ cm \times 7\ cm} = 8\ cm$$

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Total surface area of solid cuboid $= 2\,[(\text{length} \times \text{breadth}) + (\text{breadth} \times \text{height}) + (\text{height} \times \text{length})]$

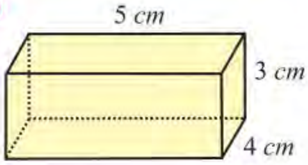
$$= 2\,\{(10 \times 7) + (7 \times 8) + (8 \times 10)\}$$
$$= 2\,[70 + 56 + 80]$$
$$= 2 \times 206$$
$$= 412\ cm^2$$

Hence, the total surface area of solid cuboid is $412\ cm^2$.

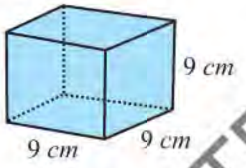
Exercise - 8.3

1. Find the volume and surface area of the following figures:

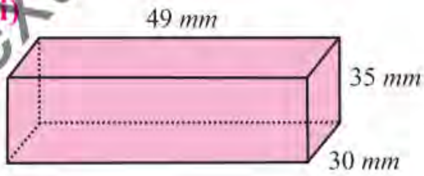
(i)



(ii)



(iii)



2. Complete the following table for each solid shape:

	Length	Breadth	Height	Surface area	Volume
(i)	$7 cm$	$8\ cm$	$8\ cm$		
(ii)		$10\ cm$	$10\ cm$		$950\ cm^3$
(iii)	$18\ cm$		$20\ cm$		$1080\ cm^3$
(iv)	$7\ cm$	$16\ cm$		$200\ cm^2$	
(v)	$15\ cm$		$15\ cm$		$2025\ cm^3$
(vi)		$17\ m$	$17\ m$	$1734\ m^2$	
(vii)	$11\ mm$		$11\ mm$		$1331\ mm^3$

3. A container is 50 *m* long, 20 *m* wide and 25 *m* high. Find the capacity of the container.

4. The inside of a cubical box measures 15 *cm* on each edge. How many cubes each measuring 1*cm* on each edge will fit inside the box?

5. Find the total surface area of a solid cube of volume $8000\ cm^3$.

6. How much water can a cubical water tank contain if it is 6 *m* long, 3 *m* wide and 5 *m* high?

7. A wall has to be built with length 19 *m*, thickness 3.5 *m* and height 9 *m*. What is the volume of the wall in cubic metres?

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8. Hamza wants to wrap a gift pack in a box for his brother. The top and bottom of the box are 32 *cm* by 15 *cm*. The sides are 9 *cm* by 8 *cm*, and the front and back are also 9 *cm* by 8 *cm*. How much wraping paper will be used to wrap the gift pack?

9. A metal cupboard is 3 *m* long 400 *cm* high and 1000 *mm* wide. What is the cost of painting the cupboard at the rate of Rs. 100 *per* m^2.

Project - 3 Measure the length, breadth and height of your geometry box and calculate the total surface area and volume.

Review Exercise 8

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1. Each of the following question is followed by four suggested options. In each case select the option.

(i) To calculate the length of boundary we use the formula of:
a. volume b. area c. perimeter d. cube

(ii) The measurement of region enclosed by any two dimensional closed figure is called:
a. perimeter b. volume c. area d. cube

(iii) The sum of all sides of rectangle is called:
a. volume b. area c. perimeter d. cuboid

(iv) The formula to calculate the breadth of each opposite side of rectangle is:
a. $\ell = b + \frac{P}{2}$ b. $b = \frac{P}{2} - \ell$ c. $P = 2(\ell + b)$ d. $A = \frac{1}{2}(\ell \times b)$

(v) If the area of rectangle is given, then length of rectangle can be calculated by the formula:
a. $b = \frac{A}{\ell}$ b. $\ell = \frac{A}{b}$ c. $b = \frac{\ell}{A}$ d. $\ell = \frac{b}{A}$

(vi) If the area of rectangle is given, then breadth of rectangle can be calculated by the formula:
a. $b = \frac{A}{\ell}$ b. $\ell = \frac{A}{b}$ c. $b = \frac{\ell}{A}$ d. $\ell = \frac{B}{A}$

(vii) Altitude is also known as:
a. hypotenuse b. diagonal c. base d. perpendicular height

(viii) The formula of area of parallelogram is same as the formula of area of:
a. square b. rectangle c. triangle d. circle

(ix) When the area of trapezium is given, the height of trapezium can be calculated by using the formula:
a. $H = 2A(s_1 + s_2)$ b. $H = \frac{2A}{s_1 + s_2}$ c. $H = \frac{s_1 + s_2}{2A}$ d. $H = \frac{2(s_1 + s_2)}{A_1}$

(x) The area of trapezium is:

a. $\frac{1}{3} \div$ (sum of parallel sides) $\times$ perpendicular height.

b. $\frac{1}{2} +$ (sum of parallel sides) $\times$ perpendicular height.

c. $\frac{1}{2} \times$ (sum of parallel sides) $\times$ perpendicular height.

d. $\frac{1}{2} -$ (sum of parallel sides) $\times$ perpendicular height.

(xi) The area of triangle can be calculated by formula:

a. $A = \ell \times b$ b. $A = \frac{1}{2}(\ell \times h)$ c. $A = 2(\ell + b)$ d. $A = \frac{1}{3}(\ell \times b)$

(xii) The half of the area of rectangle is called the area of:

a. square b. rectangle c. triangle d. circle

(xiii) Your textbook of Mathematics is a:

a. 1−D object b. 2−D object c. 3−D object d. circle shaped

(xiv) The sum of the areas of all the faces of any three -dimensional solid is known as its:

a. circumference b. surface area c. volume d. diameter

(xv) One and the only dimension-less figure is:

a. circle b. point c. triangle d. rectangle

(xvi) If the volume of any solid cube is given, we can find the length of its each side by using the formula:

a. $\ell = \sqrt[3]{V}$ b. $\ell = \sqrt[4]{V}$ c. $\ell = \sqrt{V}$ d. $\ell = \sqrt[\frac{1}{5}]{V}$

(xvii) If the total surface area of any solid cube is given we can find the length of its each side by using the formula:

a. $\ell = \sqrt[3]{\frac{S}{6}}$ b. $\ell = \sqrt{\frac{S}{6}}$ c. $\ell = \sqrt[\frac{1}{3}]{\frac{S}{6}}$ d. $\ell = \sqrt[\frac{1}{2}]{\frac{S}{6}}$

(xviii) The volume of cuboid can be calculated by using the formula:

a. $V = 4\ell^2$ b. $V = \ell^3$ c. $V = \ell \times b \times h$ d. $V = \sqrt[3]{\ell}$

(xix) The total surface area of the cube can be calculated by using the formula:

a. $S = 6\ell^2$ b. $S = \ell \times b \times h$ c. $S = \frac{\ell^2}{6}$ d. $\ell = \sqrt{\frac{5}{6}}$

(xx) If the length of an edge of a solid cube is 1 m, the volume of the solid cube is:

a. 1 m^3 b. 2 m^3 c. 3 m^3 d. 4 m^3

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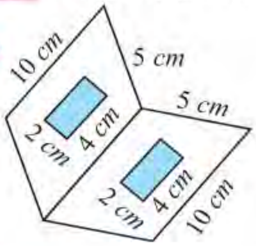
2. Define the following:

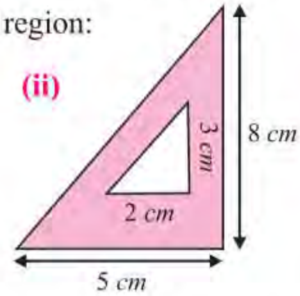
(a) Area **(b)** Perimeter **(c)** Volume of cube
(d) Volume of cuboid **(e)** Surface area of cube **(f)** Surface area of cuboid

3. Find the area of each shaded region:

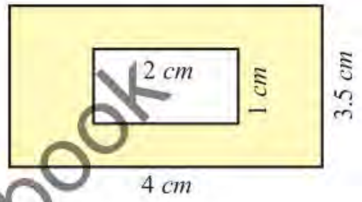
(i)



(ii)

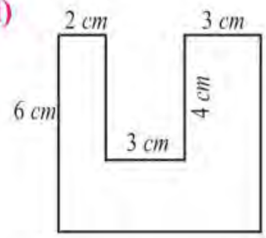


(iii)

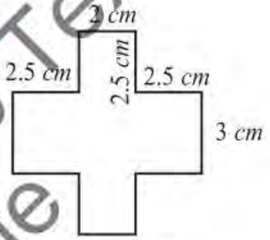


4. Find the area and perimeter of the following shapes.

(i)



(ii)



5. Namra and Momina go for jogging every morning. Namra covers a distance around a square of length 300 *m* and Momina goes around a rectangular field with length and breadth of 250 *m* and 200 *m* respectively. Who covers more distance Namra or Momina?

6. Gulzar spent Rs. 250 per square metre to pave the road in the garden. The garden is 150 *m* long and 9 *m* wide and the road is 3 *m* wide. How much money did he spend in all?

7. Nadia cuts a 30 *cm* by 5 *cm* cloth into 6 equal parts. Find the area of each piece.

8. Zahoor wants to paint the main gate of his house with white colour. The dimensions of the gate are 10 feet and 12 feet. How much money Zahoor needs to spend for painting of his house gate at the cost of Rs. 500 per square foot?

9. A match box measures 4 *cm* × 2.5 *cm* × 1.5 *cm*. What will be the volume of packet containing 12 such boxes?

10. Waqas wants to wrap a gift in a box for Ahmad. The top and bottom of the box are 21 *cm* by 8 *cm* respectively. The sides are 8 *cm* by 6 *cm* and the front and back are 8 *cm* by 6 *cm*. How much wrapping paper will be needed to wrap the gift?

11. Waqas and Saba are building a cuboid shaped room for their daughter. If the room is 6 feet high, 10 feet wide and 8.5 feet long then how much space will there be inside?

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Summary

- The distance around any two dimensional closed shape is called perimeter.
- The region enclosed within the boundary of a closed shape is called area.
- The perimeter of square can be calculated by using the formula:
  Perimeter = 4 × side length = 4ℓ
- The area of square can be calculated by using formula:
  Area = side length × side length = $\ell \times \ell = \ell^2$
- The perimeter of rectangle can be calculated by using formula:
  Perimeter = 2 (length + breadth) = $2(\ell + b)$
- The area of rectangle can be calculated by using formula:
  Area = length × breadth = $(\ell \times b)$
- The area of path can be calculated by using formula:
  Area of path = Area of larger shape − Area of smaller shape
- The altitude is also known as perpendicular height.
- A parallelogram is a 4-sided closed shape formed by two pairs of parallel lines.
- Area of parallelogram can be calculated by using the formula:
  Area = base length × perpendicular height
- A trapezium is a quadrilateral with four sides and four angles. It has only two parallel lines.
- If any trapezium has only one pair of parallel equal sides in measurement the trapezium is called regular trapezium.
- The area of trapezium can be calculated by using the formula:
  $$\text{Area} = \frac{1}{2}(\text{base}_1 + \text{base}_2) \times \text{Perpendicular height} = \frac{1}{2}(S_1 \times S_2) \times h$$
- The area of triangle can be calculated by using the formula:
  $$\text{Area} = \frac{1}{2}(\text{side} \times \text{side}) = \frac{1}{2}(b \times h)$$
- The volume is a quantity or amount of space enclosed by a closed surface or container.
- The surface area of a solid figure is the sum of the area of all its faces.
- 3-dimensional figures/shapes are the solid figures which have thickness and can be held in human hand.
- A 3-dimensioanl solid body bounded by six square surfaces is known as cube.
- A 3-dimensional solid body bounded by six rectangular surfaces is known as cuboid.
- The surface area of cuboid can be calculated by using the formula:
  $$S = 2[(\ell \times b) + (b \times h) + (h \times \ell)]$$
- The surface area of cube can be calculated by using the formula: $S = 6\ell^2$
- The volume of cuboid can be calculated by using the formula: $V = \ell \times b \times h$
- The volume of cube can be calculated by using the formula: $V = \ell^3$

Go Online

Use the following online game links to practice volume of cube and cuboid.
https://www.splashlearn.com/s/math-games/find-volume-using-the-formula
https://www.splashlearn.com/s/math-games/solve-the-word-problems-related-to-volume

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Sub-Domain 9

SYMMETRY

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Identify 2-D, 3-D shapes with respect to their characteristics.
- Differentiate between parallel and intersecting lines.
- Identify transversal angles related to them and recognize rotational symmetry.

Allah used beautiful Mathematics in creating the world. How can you identify the symmetrical lines in leaves, butterfly and a flower.

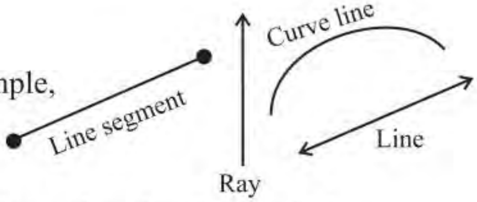
9.1 IDENTIFICATION OF 2-D AND 3-D SHAPES WITH RESPECT TO THEIR CHARACTERISTICS

We are familiar about the different types of geometrical shapes such as line, rays, triangles, rectangles and other different types like polygons, etc. Some of them are one-dimensional figures and some are two-dimensional figures. We have learnt about these figures in our previous classes.

1-Dimensional figures

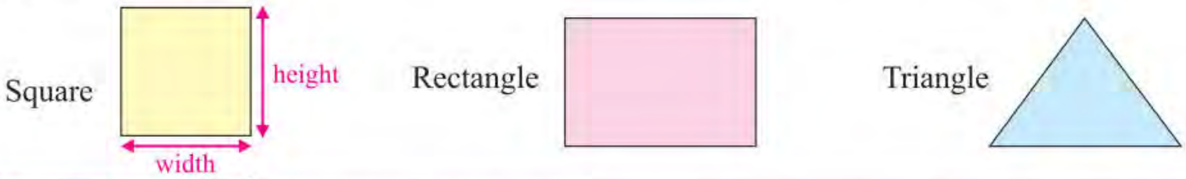
The figures that have only length, these figures do not have width or thickness.

For example,



2-Dimensional figures

The figure that can be drawn on paper. These figures have no thickness but they have width and height. For example,



Teaching Point Encourage students to ask questions at any stage during learning process. Students questioning serves a wide variety of purposes. e.g.

- to assess their understanding.
- to keep learners engage during an explanation.
- to develop their thinking or focus their attention on topic.

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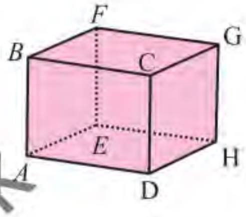
3-Dimensional figures

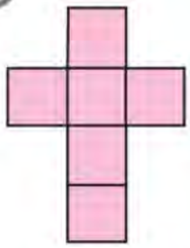
The solid figures are 3-dimensional figures. These figures have height, width and depth. All solid figures have thickness and can be held in our hand. For example,

i. Cube

A 3-dimensional solid body bounded by six square surfaces is known as cube.

A cube and its net is given in the figure.



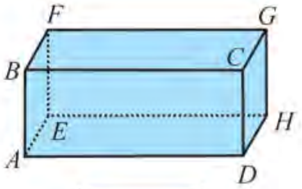


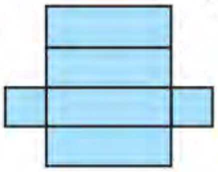
- **Square surfaces:** **(i)** *ABCD* and *EFGH* (opposite surfaces)
  (ii) *ABFE* and *DCGH* (opposite surfaces)
  (iii) *ADHE* and *BCGF* (opposite surfaces)
- **Vertices:** There are 8 vertices of cube, i.e. *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*
- **Edges :** There are 12 edges of a cube, i.e.
  $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{BF}$, $\overline{FG}$, $\overline{GC}$, $\overline{GH}$, $\overline{HE}$, $\overline{HD}$, $\overline{AE}$ and $\overline{EF}$

ii. Cuboid

A 3–dimensional solid body bounded by six rectangular surfaces is known as cubiod.

A cuboid and its net is given in the figure.

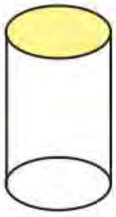




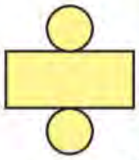
- **Rectangular Surfaces:** **(i)** *ABCD* and *EFGH* (opposite surfaces)
  (ii) *ADHE* and *BCGF* (opposite surfaces)
  (iii) *ABFE* and *DCGH* (opposite surfaces)
- **Vertices:** There are 8 vertices of a cuboid i.e. *A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*
- **Edges:** The are 12 edges of a cuboid i.e.
  $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{BF}$, $\overline{FG}$, $\overline{GC}$, $\overline{GH}$, $\overline{HE}$, $\overline{HD}$, $\overline{AE}$ and $\overline{EF}$

iii. Cylinder

A 3–dimensional solid body with two circular surfaces of equal radius and one curved surface between them is known as cylinder.



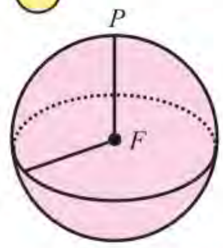
A cylinder and its net is given in the figure.



- **Surfaces:** A cylinder has 2 circular and 1 curved surface.
- **Vertices:** A cylinder has no vertex.
- **Edges:** It has 2 edges.

iv. Sphere

A 3–dimensional solid body with completely round surface whose all points on its outer surface are at an equal distance from a fixed point is known as sphere.



A sphere is given in the figure. The fixed point is called centre (*F*) and distance ($m\overline{PF}$) is called radius of the sphere.

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Note The fixed point "*P*" is called the centre of the sphere.

iv. Hemisphere

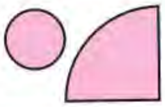
A 3-D shape made from cutting a sphere into two equal halves with one flat side is known as hemisphere.

- **Surfaces:** It has one flat and one curved surface.
- **Vertices:** It has no vertex.
- **Edges:** It has one curved edge.

v. Cone

A 3-dimensional solid body with one circular surface and a slanting curved surface is known as cone.

A cone and its net is given in the figure.



- **Surfaces:** A cone has two surfaces.
- **Vertices:** A cone has only one vertex.
- **Edges:** A cone has only one edge.

Note The distance between the centre of the base of cone and its vertex is called height of the cone. But the distance of the vertex from any point on the boundary is known its slant height.

Activity -1 Observe in your surrounding and write the names of at least two real life objects that resemble with cube, cuboid, cylinder, sphere, cone, rectangle, circle and hemisphere.

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Exercise -9.1

1. Write the name of 3-dimensional figures which resemble to these objects:

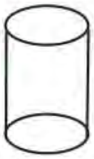
(i) (ii)

(iii) (iv) (v) (vi)

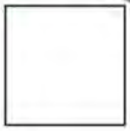
2. Sort 1-D, 2-D and 3-D shapes.

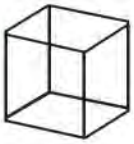
(i)
Cone

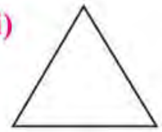
(ii)
Rectangle

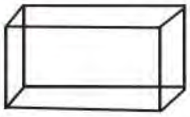
(iii) 
Cylinder

(iv)
Line

(v) 
square

(vi) 
Cube

(vii) 
Triangle

(viii) 
Cuboid

3. Write down the edges, surfaces and vetices of the following:

(i) cylinder (ii) cuboid (iii) cube (iv) cone (v) hemisphere

Teaching Point

Bring the original 2-D and 3-D objects in the classroom to explain the concept with their characteristics.

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Interesting Information Our brain can be divided into left side and right side which are called the left hemisphere and the right hemisphere. Information passes between the two hemispheres by a bundle of nerves and fibers making them work as a single unit.

Practice -1 Prepare a chart paper for all 2-D and 3-D geometrical figures and relate them with corresponding real life object (at least one).

9.2 PARALLEL LINES, INTERSECTING LINES, PERPENDICULAR LINES AND TRANSVERSAL

i. Parallel lines

The railway track and zebra crossing never intersect each other at any point.

"Two or more lines which extends in the same direction and remain the same distance apart are called parallel lines." e.g.

Remember

(i) Line: A line is a series of points that extends in both directions without ends. e.g., $\overleftrightarrow{AB}$

(ii) Ray: A ray is a line with one end point. e.g., $\overrightarrow{AB}$

(iii) Line segment: A line segment is a part of line that is made by joining two points together. e.g., $\overline{AB}$

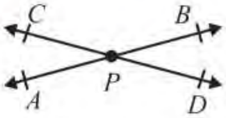
In the above figures the parallel lines are represented by the single or double arrow.
The symbol "||" is used to represent the parallel lines.

ii. Intersecting lines/non-parallel lines

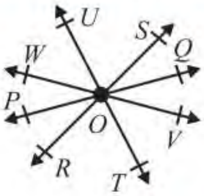
The legs of the table and hands of a scissors are crossing each other.

"Two lines that are intersecting each other at a single point are called intersecting lines." The point where these lines intersect is called point of intersection. e.g.,

(a)



(b)



In the above figures all are the intersecting lines.

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Practice -2

Identify the parallel and intersecting lines.

(i)

(ii)

iii. Perpendicular lines

Observe in your surroundings e.g.,

a

A tall tree on the ground.

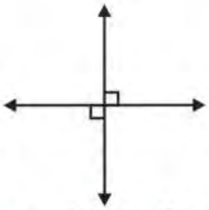
b

An electric pole or pole of our national flag on the pavement.

c

Corner of two adjacent walls and high buildings.

"Two or more lines that meet or intersect each other at right angle (90°) are called perpendicular lines." e.g.,

(a)


Perpendicular lines

(b)

Perpendicular lines

Fun Fact

Perpendicular lines are always intersecting lines but intersecting lines may or may not be perpendicular.

iv. Transversel

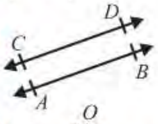
Observe the road crossing on the railway track.

"A line that passes through two or more lines at two distinct points is called transversel".

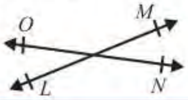
A line *PQ* passing through the pair of parallel lines AB and *CD*. The line *PQ* is called transversel.

Practice -3

Identify the perpendicular lines.

(i)


(ii)

(iii)


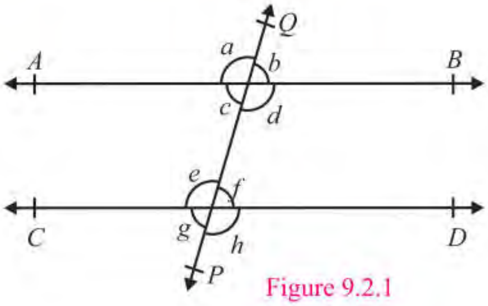


Figure 9.2.1

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Transversel angles

When a transversal intersects two parallel or non parallel lines, it forms different kinds of angles. These angles can be categorized into several types:

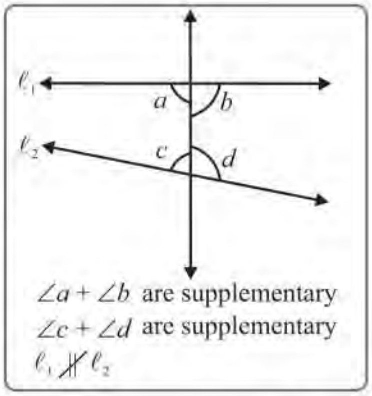
- Interior angles.
- Exterior angles.
- Pair of corresponding angles.
- Pair of alternate interior angles.
- Pair of alternate exterior angles.
- Pair of interior angle on the same side of transversal.

Let's understand the angles from figure 9.2.1 made by transversal.

Sr.No.	Type of angles	Angles
1	Interior angles	$\angle c, \angle d, \angle e, \angle f$
2	Exterior angles	$\angle a, \angle b, \angle g, \angle h$
3	Pairs of corresponding angles	$\angle a$ and $\angle e$, $\angle b$ and $\angle f$ $\angle c$ and $\angle g$, $\angle d$ and $\angle h$
4	Pairs of alternate interior angles	$\angle c$ and $\angle f$ $\angle d$ and $\angle e$
5	Pairs of alternate exterior angles	$\angle a$ and $\angle h$ $\angle b$ and $\angle g$
6	Pairs of interior angles on the same side of transversal	$\angle c$ and $\angle e$ $\angle d$ and $\angle f$

From the above discussion, we conclude that the angles associated with the transversal will have the following properties:

(i) The corresponding angles are equal in measurement.
(ii) The alternate angles are equal in measurement.
(iii) The interior angles are supplementary.



The converse statement for above properties are also true. e.g.

(i) If the corresponding angles are equal, then the two lines are parallel.
(ii) If the alternate angles are equal, the two lines are parallel.
(iii) If the interior angles are supplementary then the two lines are parallel.

Vertically opposite angles

"When two lines intersect at a point then the opposite angles are formed at the point of intersection. These angles are called vertically opposite angles."
In the figure 9.2.2, $\angle a$ and $\angle d$, $\angle b$ and $\angle c$ are vertically opposite angles.

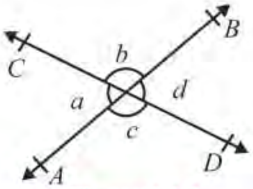


Figure 9.2.2

Remember A pair of vertically opposite angles are always equal to each other.

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Example 1 If the measure angle $a = 60°$ then find the values of other angles on the intersecting lines AB and CD.

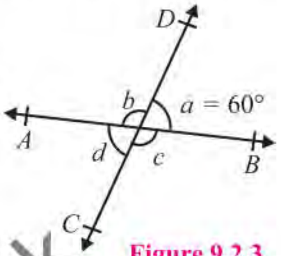


Figure 9.2.3

Solution As, vertically opposite angles are equal so, $m\angle a = m\angle d = 60°$.

Since, $\angle a$ and $\angle b$ are supplementary. So,

$$m\angle a + m\angle b = 180°$$
$$60° + m\angle b = 180°$$
$$m\angle b = 180° - 60°$$
$$m\angle b = 120°.$$

Now, by applying the property of vertically angles

$$m\angle b = m\angle c = 120°.$$

Challenge

If the value of the largest angle made by intersecting lines is 109°. Then find the other angles.

Example 2 Calculate the value of unknown angles in the figure.

Solution In the figure, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel lines T_1 and T_2 are the transversals.

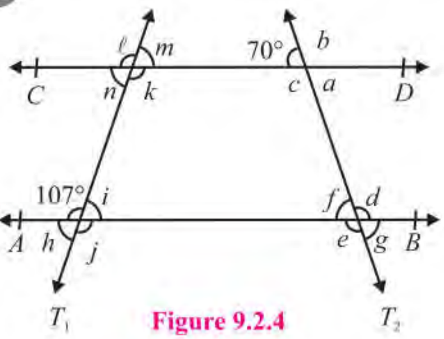


Figure 9.2.4

For transversal T_1

$m\angle j = 107°$ (vertically opposite angles)

$m\angle i + 107° = 180°$ (adjasent angles on a line are supplementary)

$m\angle i = 180° - 107°$

$m\angle i = 73°$

$m\angle i = m\angle h = 73°$ (vertically opposite angles)

Now, $m\angle i = m\angle n = 73°$ (alternate angles are equal)

and $m\angle k = 107°$

$m\angle k = m\angle \ell = 107°$ and $m\angle n = m\angle m = 73°$ (vertically opposite angles are equal)

For transversal T_2

$m\angle a = 70°$ (vertically opposite angles are equal)

$m\angle a + m\angle b = 180°$ (adjasent angles on a line are supplementary)

$70° + m\angle b = 180°$

$m\angle b = 180° - 70°$

$= 110°$

So, $m\angle b = m\angle c = 110°$ (vertically opposite angles are equal)

Now, $m\angle b = m\angle d = 110°$, (corresponding angles are equal)

$m\angle d = m\angle e = 110°$, (vertically opposite angles)

$m\angle a = m\angle g = 70°$ and $m\angle f = 70°$ ($m\angle f = m\angle g$)

Activity -1

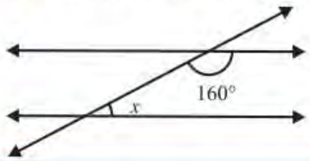
Make a comparison between the properties of parallel lines and intersecting/perpendicualr lines. Differentiate them by giving real life examples from your surroundings.

Go Online Visit the following links to play games online.

http://www.mathgames.com/skill/8.46-transversal-of-parallel-lines
http://www.transum.org/software/SW/starter_of_the_day/students/angleparallel.asp

Practice -4

Find the value of x in the figure.



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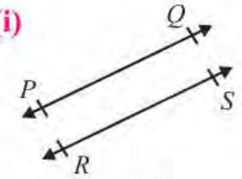
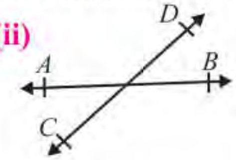
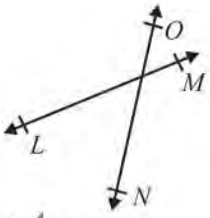
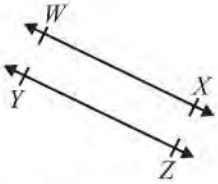
Exercise - 9.2

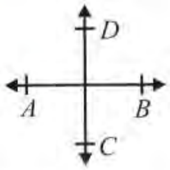
1. Identify the parallel, intersecting and perpendicular lines:

(i)


(ii)


(iii)


(iv)


(v)


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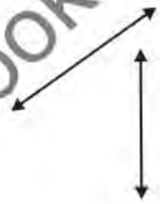
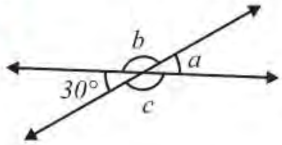
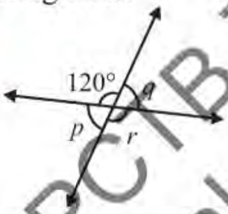


2. Find the unknown angles of the intersecting lines:

(i)


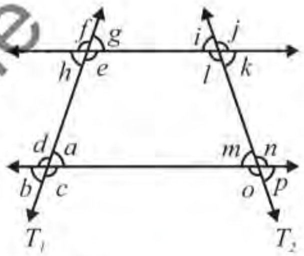
(ii)


3. Observe the diagram and write all:

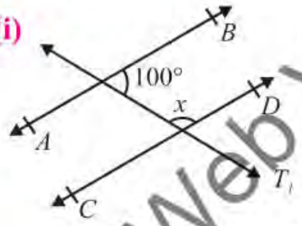
(i) Pair of corresponding angles

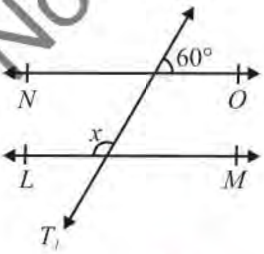
(ii) Pair of alternate angles

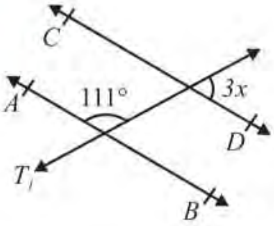
(iii) Exterior angles

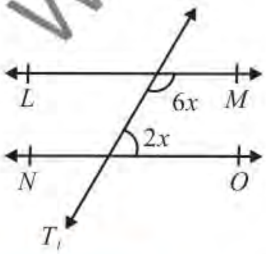


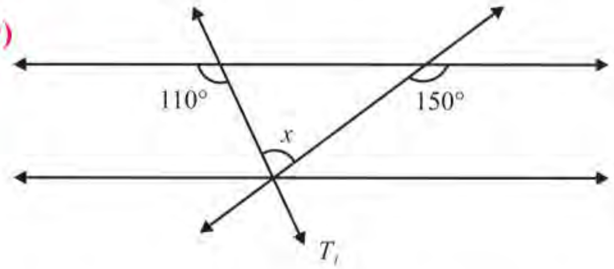
4. Calculate the values of unknown angles in the following diagrams:

(i)


(ii)


(iii)


(iv)


(v)


Project - 2

Draw a transversal for non-parallel lines. Does these transversal hold the same properties that holds for parallel lines? If yes, then prepare a chart for all the properties, also give an example from your surroundings.

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9.3 SYMMETRY

Symmetry is all around us in nature and in the field of art and design.

It is defined as, "one shape is identical to the other shape when it is moved, rotated or flipped,. then we say this shape is in symmetrical shape." In our previous class, we have learnt about symmetry and the line of symmetry. e.g.

"A line that divides an object into two identical pieces, is called line of symmetry." Here, in this section, we will learn more about symmetry.

9.3.1 Types of Symmetry

Here, we will discuss about two types of symmetry:

(i) Reflective symmetry **(ii)** Rotational symmetry

i. Reflective symmetry

"If a shape or pattern is reflected in a mirror line or line of symmetry then this type of symmetry is called reflective symmetry". For example,

a The reflection of mountain in the lake.

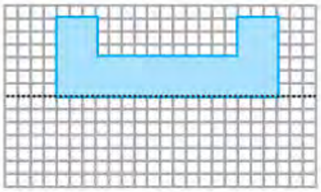
b The left side is reflection of right side.

In the above figures, the mirror line divides figures into two parts.

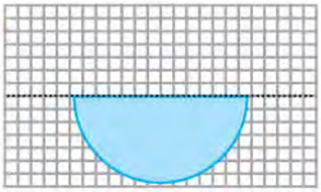
Example 3 Complete the following shapes according to the given mirror line or line of symmetry.

(i)

(ii)



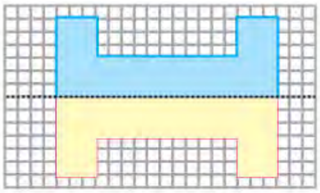
(iii)



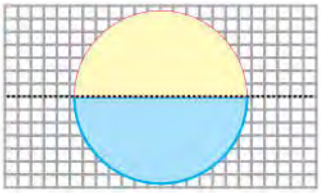
Solution Draw the same shape on the other side of mirror line or line of symmetry. Simply, flip the shapes and complete them.

(i)

(ii)



(iii)



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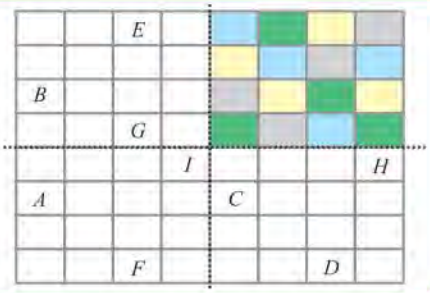
Practice -5 Look at the pattern in the grid. If the pattern is reflected in the two lines of symmetry. What colour will these squares be?

A *B* *C*

D *E* *F*

G *H* *I*

Copy the pattern and colour the rest of the corresponding squares, when it is reflected in the mirror lines.



Activity -2 Fold a piece of paper in such a way that each part is equal to other part and identify mirror line.

ii. Rotational symmetry

Observe in your surroundings, some of the shapes you see will have line of symmetry but some of the shapes you see, looks the same as you turn them e.g.

a

Pedestal fan.

b

Wheels of tractor.

c

Circular table.

"If a shape rotated about a point to another position and still looks the same then the shape has rotational symmetry."

In the above figures all the shapes above have rotational symmetry.

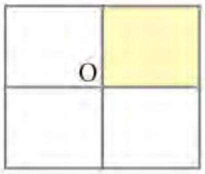
"Number of times a shape looks the same in one full turn is called order of symmetry."

For example

This rectangle and the letter "S" has rotational symmetry of order 2.

Example 4

Rotate the square and find the order of symmetry.

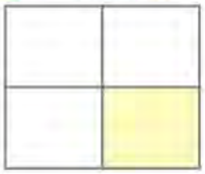


Remember

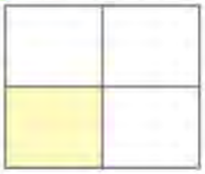
A fixed point around which the rotation ocurs is called the centre of rotation.

Solution

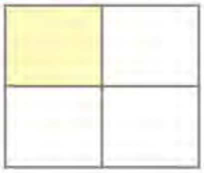
a 1st rotation



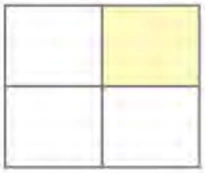
b 2nd rotation



c 3rd rotation



d 4th rotation



At 4th rotation figure came on its original position. So, the order of symmetry is 4 around 90°.

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Go Online Visit the following links and play the online games.
https://www.mathgames.com/skill/8.73-reflections-graph-the-image
Visit the below link for more practice about rotational symmetry.
https://www.mathsisfun.com/geometry/symmetry-rotational.html

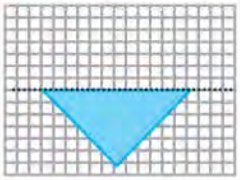
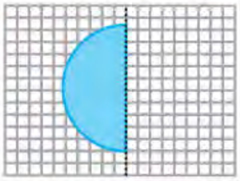
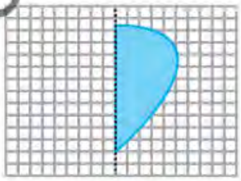
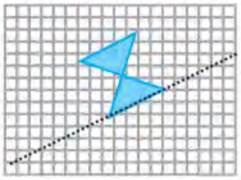
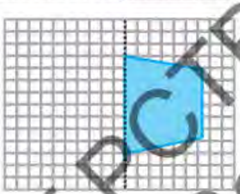
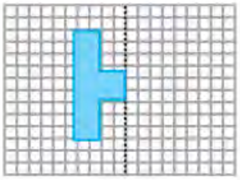
Practice -6

Draw the following shapes and rotate each figure at 90°. Also, write the order of symmetry for each shape.

- Circle
- Rectangle
- Rhombus
- Kite

Exercise - 9.3

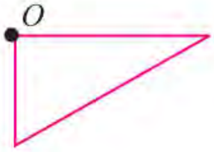
1. Use the concept of reflective symmetry to complete the following shapes:

(i) 
(ii) 
(iii) 
(iv) 
(v) 
(vi) 

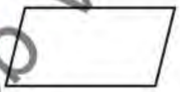
2. Rotate the following shapes and write the order of symmetry:

(i)

(ii)

(iii) 

3. Draw the possible lines of symmetry for the following shapes

(i)
(ii) 
(iii)
(iv)
(v)
(vi)

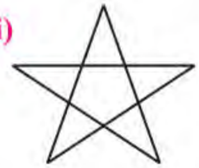
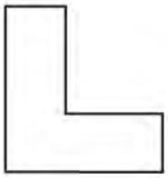
4. Draw a shape with exactly two lines of symmetry.

5. Draw a shape with exactly 6 lines of symmetry.

6. Draw a shape with exactly 3 order of rotation.

7. Draw a shape with exactly 6 order of rotation.

8. Find the order of rotational symmetry for the following shapes.

(i)
(ii) 
(iii) 

Project - 3

Observe in your surrounding and make a list of at least four items that have a line of symmetry. Also, draw them and write order of rotational symmetry.

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Review Exercise 9

1. Each of the following questions is followed by four suggested options. In each case select the correct option.

(i) A sphere is a 3-dimensional solid object. It has:
- a. 6 surfaces, 12 edges, 8 vertices
- b. 0 surfaces, 0 edges, 1 vertices
- c. 0 surfaces, 0 edges, 0 vertices
- d. 1 surface, 0 edges, 0 vertices

(ii) A cuboid has:
- a. 4 faces
- b. 6 faces
- c. 12 faces
- d. 8 faces

(iii) A hemisphere has
- a. 0 edges
- b. 1 edge
- c. 2 edges
- d. 4 edges

(iv) Two lines that never intersect each other at any point are called:
- a. perpendicular lines
- b. intersecting lines
- c. transversal lines
- d. parallel lines

(v) Perpendicular lines always intersect each other at:
- a. 20°
- b. 45°
- c. 90°
- d. 120°

(vi) A point where two lines intersect each other is called:
- a. corner point
- b. centre point
- c. point of intersection
- d. none of these

(vii) A line that passes through two or more parallel lines at distinct points is called:
- a. perpendicular
- b. transversal
- c. altitude
- d. hypotenuse

(viii) If the value of largest angle made by intersecting lines is 120°, then the value of its opposite angle will be:
- a. 60°
- b. 50°
- c. 110°
- d. 120°

(ix) Line that divides an object into two indentical pieces is called:
- a. perpendicualr line
- b. mirror line
- c. segment
- d. ray

(x) Number of times a shape looks the same in one full turn is called:
- a. symmetry
- b. center of symmetry
- c. power of symmetry
- d. order of symmetry

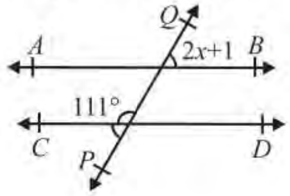
2. Define the following:

(i) Parllel lines (ii) Non-parallel lines/ intersecting lines (iii) Perpendicular lines
(iv) Transversal (v) Vertically opposite angles (vi) Symmetry
(vii) Reflective symmetry (viii) Rotational symmetry

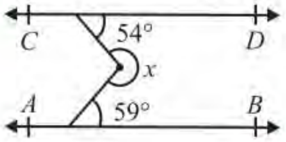
3. Differentiate between 2-D and 3-D shapes and give one example for each one.

4. Find the value of x in the following figures:

(i)



(ii)



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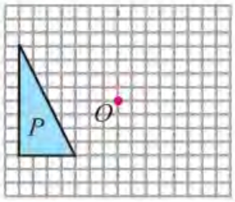
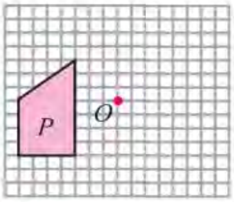
5. In each of the following questions, copy the grid and the object *P*. Draw the image *Q* when the object is rotated by the angle stated, about the centre of rotation *O*:

(i)



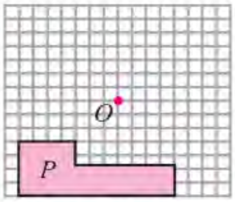
Rotation by 90° clockwise about *O*

(ii)



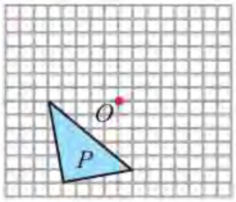
Rotation by 180° about *O*

(iii)



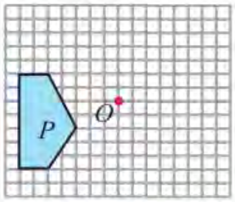
Rotation by 180° about *O*

(iv)



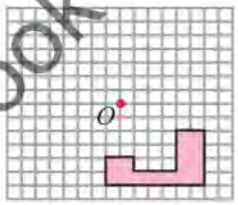
Rotation by 90° anti-clockwise about *O*

(v)



Rotation by 180° about *O*

(vi)



Rotation by 90° anti-clockwise about *O*

Summary

- The figures that have only length are called 1-D figures.
- The figures that can be drawn on paper, these figures have no thickness but they have length and width/height. These figures are called 2-D figures.
- The figures that have length, width and depth are called 3-D figures.
- Two or more lines which extend in the same direction and remain the same distance apart are called parallel lines.
- Two lines that are intersecting each other at a single point are called intersecting lines. The point where these lines intersect, is called point of intersection.
- Two lines that meet or intersect each other at right angle (90°) are called perpendicular lines.
- A line that passes through two or more lines at distinct points is called transversal.
- When two lines intersect at a point then the opposite angles are formed at the point of intersection. These angles are called vertically opposite angles.
- A line that divides an object into two identical pieces is called line of symmetry.
- If a shape or a pattern is reflected in a mirror line or line of symmetry then this type of symmetry is called reflective symmetry.
- If a shape is rotated about a point to another position and still looks the same then the shape has rotational symmetry.
- Number of times a shape looks the same in one full turn is called order of symmetry.

Teaching Point The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

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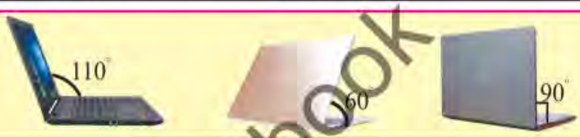
Sub-Domain 10 GEOMETRICAL CONSTRUCTIONS

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Construct angles of different measures.
- Define bisectors of an angle, bisector and perpendicular bisector of a line segment.

Which is the maximum angle that can be formed by opening your laptop screen?

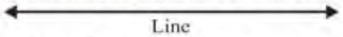


INTRODUCTION

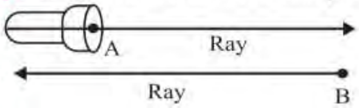
We are familiar with the geometry, it is a branch of Mathematics. The word geometry comes from the Greek that means "measurement of the earth." It concernes with the properties of points, straight lines and curved lines, and surface of solids. "A line segment is a part of a line that is bounded by two distinct end points and contains every point on the line between its end points." The part from A to B of the above line is known as line segment. It is denoted by $\overline{AB}$.

Remember

- **Line:** A line is a series of points that extends in both directions without ends.



- **Ray:** A ray is a line with one end point.



10.1 CONSTRUCTION OF LINE SEGMENT

We can draw a line segment by using compass and ruler. Look at the following examples.

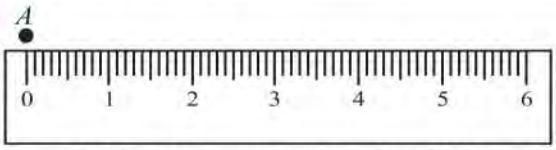
Example 1 Draw a line segment $m\overline{AB} = 4\ cm$ using ruler.

Construction: To draw a line segment simply follow the steps below.

Step I

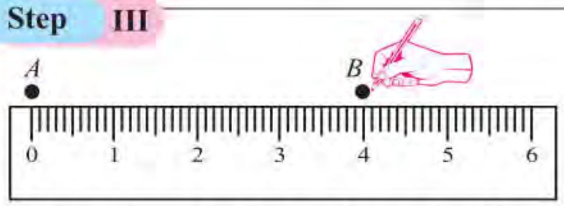
Mark a point A on a paper.

Step II



Place a ruler such that its zero point should be attached with point A.

Step III



Mark another point B, $4cm$ from point A by using ruler.

Step IV

Now, join points A and B together and then remove the ruler.

$m\overline{AB} = 4\ cm$ is the required line segment.

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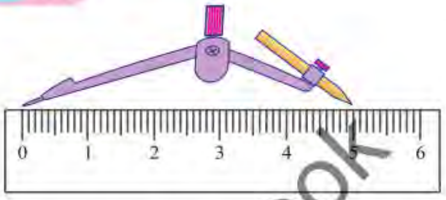
Example 2 Draw a line segment of 5 *cm* using compass.

Contraction: To draw a line segment simply follow the below steps.

Step I

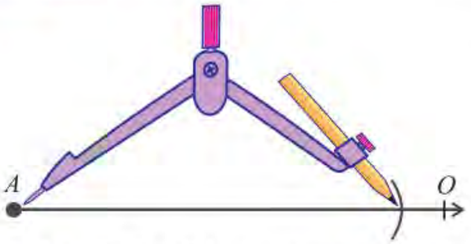
Draw a ray *AO* on a paper.

Step II



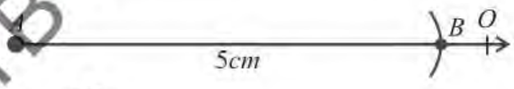
Place the needle of compass at point '*O*' using ruler and open it 5 *cm*, as shown above.

Step III



Place the needle of the compass at point *A*, then draw an arc 5 *cm* on $\overrightarrow{AO}$.

Step IV



The arc cuts $\overrightarrow{AO}$ at *B* so, $m\overline{AB} = 5\ cm$ is a required line segment.

Practice -1

(i) Use your ruler to draw a line segment of 6cm.

(ii) Use your compass to draw a line segment of 9cm.

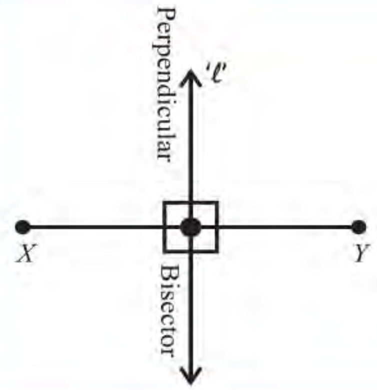
10.2 PERPENDICULAR BISECTOR OF A LINE SEGMENT

'Bi' means two and 'section' means part so, bisection means divide into two equal parts.

The perpendicular bisector of a line segment is a line which makes a right angle with given line segment and divide it into two equal parts.

A perpendicular bisector always passes through the mid point of the line segment. It can be constructed by using a pair of compass and ruler.

Now, look at the following example to draw the perpendicular bisector of a given line segment.



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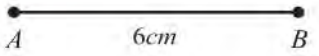
Example 3 Draw a perpendicular bisector of $m\overline{AB} = 6\ cm$.

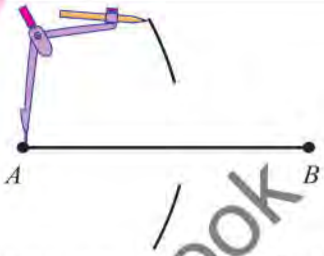
Construction: To draw perpendicular bisector simply follow the following steps.

Step I



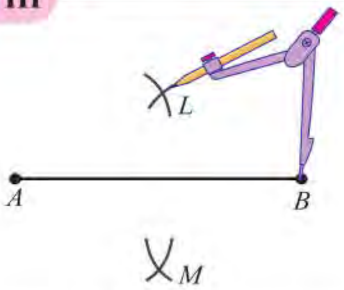
Draw $m\ \overline{AB} = 6\ cm$.

Step II



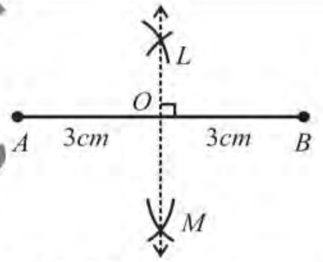
Open the compass more than half of $m\overline{AB}$. With centre at A draw two arcs on both sides of $\overline{AB}$.

Step III



With center at B draw two arcs of same radius cutting previous arcs at L and M respectively.

Step IV



Join L and M with blue line cutting $\overline{AB}$ at point O, O is a right angle.

So, $\overleftrightarrow{LM}$ is a perpendicular bisector of $\overline{AB}$ and $m\ \overline{AO} = m\ \overline{OB} = 3cm$.

Note While drawing perpendicular bisector, on notebook only draw step (iv) figure and write all steps of construction.

Remember The perpendicular bisector is also known as right bisector.

10.2.1 Difference between Bisector and Perpendicular Bisector

Bisector

Bisector is a line that divides something e.g., line segment or angle into two equal parts.

Perpendicular bisector

Perpendicular bisector is a line that bisects a given line segment at a right angle. Also, it passes through the midpoint of the line segment.

10.2.2 Construction of a Perpendicular to a Given Line from a Point on it

The method to draw a perpendicular to a given line at a given point is explained in the following example:

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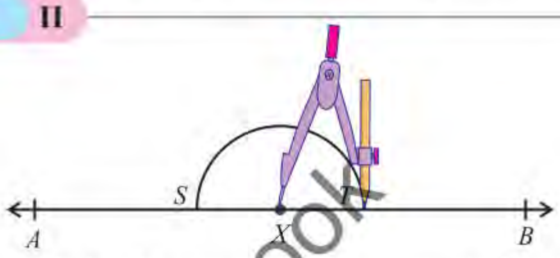
Example 4 Draw a perpendicular to $\overleftrightarrow{AB}$ at a given point on $\overleftrightarrow{AB}$.

Construction: To draw the perpendicular to $\overleftrightarrow{AB}$ at a given point, simply follow the steps below.

Step I

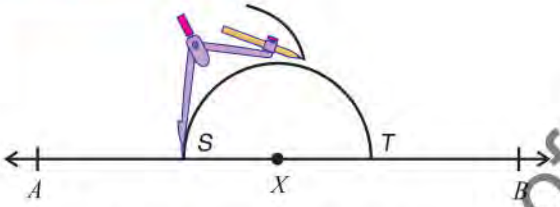
Draw a line AB of given measurement and mark point X on the line.

Step II



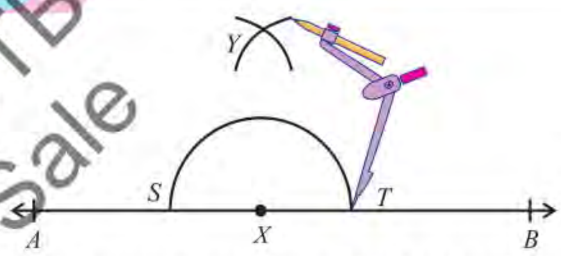
With the help of compass draw a semi-circle of any radius with centre at point 'X'. This circle touching line AB at points S and T respectively.

Step III



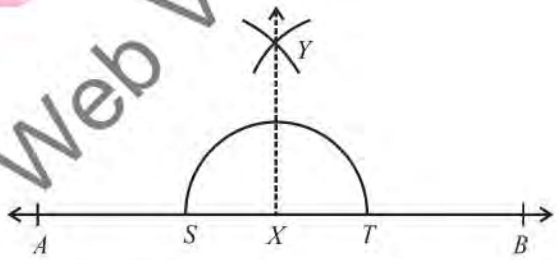
Mark an arc above the line of any radius centre at point S.

Step IV



Now mark another arc above the line with the same radius centred at point T. This arc cuts the first arc at point Y.

Step V



Now, join point Y with X. So, $\overrightarrow{XY}$ is required perpendicular to $\overleftrightarrow{AB}$ at a given point X on it.

Practice -3

Draw a line segment $AB = 5\ cm$, then mark a point P on it and draw a perpendicular at this point.

Note

While drawing bisector on notebook, only draw step (V) figure and write all steps of construction.

10.2.3 Construction of a Perpendicular to a Given Line from a Point outside the Line

The method to draw a perpendicular to a given line from a point outside the line is explained in the following example.

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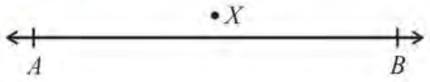
Example 5 Draw a perpendicular to the line from a point X outside the line.

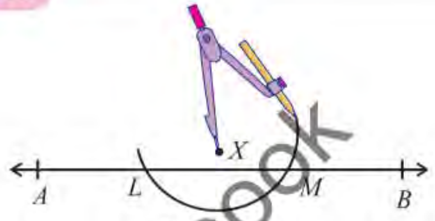
Construction: To draw perpendicular to the line from a point outside, simply follow the following steps.

Step I



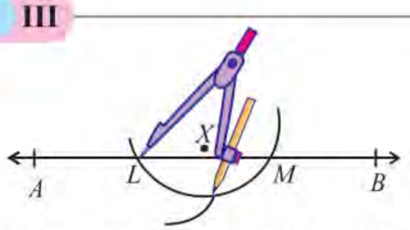
Draw $\overline{AB}$ of any measurement and mark point X above the line as shown.

Step II



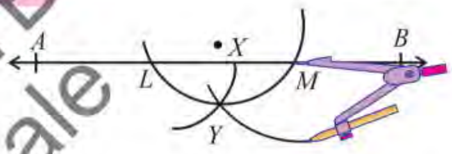
Draw an arc of any radius with centre at point X. The arc should cut the line at two points say L and M.

Step III



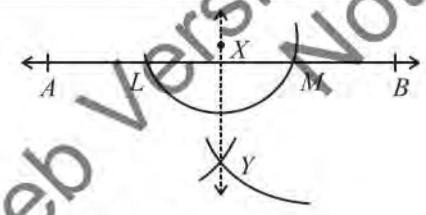
Mark second arc of same radius with centre at L.

Step IV



Mark third arc of same radius with centre at M, this arc cuts the second arc at point Y.

Step V



Join point X with Y, with line XY.

So, $\overleftrightarrow{XY}$ is required perpendicular to $\overleftrightarrow{AB}$ from a point X outside the line.

Practice -4

Draw a perpendicular bisector to the line segment $m\overline{AB} = 6cm$, from point "P" that is outside the line.

Note

While drawing perpendicular on notebook, only draw step (V) figure and write all points in construction steps.

Go Online

Visit the following link to show construction of line bisection.
https://www.mathspad.co.uk/i2/construct.php

Remember

A right bisector always intersects the given line at a right angle (90°). It can be checked by using protractor.

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Exercise - 10.1

1. Draw line segments of the following measurements using ruler and compass:

(i) 5 *cm* **(ii)** 3.9 *cm* **(iii)** 7 *cm* **(iv)** 5.7 *cm* **(v)** 6.5 *cm*

2. Draw the right bisectors of the following line segments:

(i)

(ii)

(iii)

(iv) 7.8 *cm* **(v)** 8 *cm* **(vi)** 90 *mm* **(vii)** 6.7 *cm*

3. Draw the perpendicular bisectors of the following line segments:

(i) 7 *cm* **(ii)** 6.8 *cm* **(iii)** 8 *mm* **(iv)** 92 *mm* **(v)** 5.8 *cm*

4. Draw a perpendicular from the out side point on the following line segments:

(i) 8.9 *cm* **(ii)** 7.8 *cm* **(iii)** 110 *mm* **(iv)** 7 *cm* **(v)** 9 *cm*

5. Draw a $m\overline{AB} = 8\ cm$ mark a point P outside it, on some height over $\overline{AB}$ then draw perpendicular from point X to $\overline{AB}$.

Project - 1 Make a booklet of rules/steps of construction for line segment, line bisector and perpendicular bisector.

History Euclid was a Greek Mathematician. He was the first person who introduced the term "Geometry". Because of his contribution in geometry he is known as the "father of geometry." To know more about his contribution in mathematics, go online and visit the following link. http://www.math.goodreads.com/author/show/125792.Euclid

(365BC-275BC)

10.3 CONSTRUCTION OF ANGLES

If a person rises the screen of a laptop slowly, the edge rises and creates a gap between the screen and the base. If the screen is raised higher, the two sides remain joined at one end, the space between the other two ends is increased. Moving the screen higher we get different positions.

Rising of screen from different angles between the base and the screens, so, an angle refers to the space between two surfaces or intersecting lines at the point where they meet.

In previous classes, we have learnt about the construction of angles of different measures using protractor. Now, we will learn the construction of angle by using the pair of compasses.

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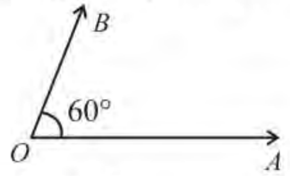
10.3.1 Construction of Angle Equal in Measure of a Given Angle

We can construct an angle equal in measure of a given angle using compass. Look at the following example.



Example 6 Construct $m\angle AOB = 60°$.

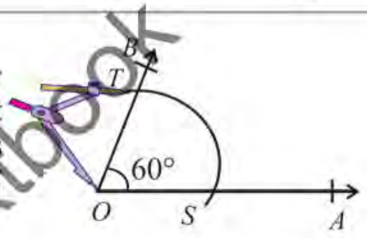
Construction:

Step I

Draw $\overrightarrow{PQ}$ of any suitable measurement as shown above.

Step II

Draw an arc of any radius with centre at point O. This arc cuts $\overrightarrow{OA}$ at point 'S' and $\overrightarrow{OB}$ at point T.

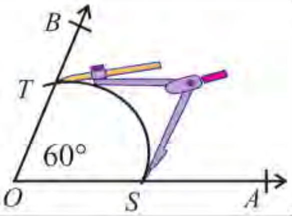


Step III

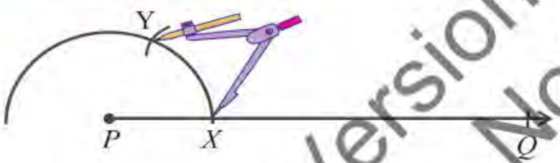
With centre 'P' draw an arc of radius equal to $m\overline{OS}$ to cut $\overrightarrow{PQ}$ at point X.

Step IV

Now, place the compass on S and take the measurement of arc ST, as given.



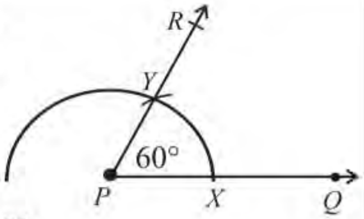
Step V



Place the needle at point X and draw another arc which cut the first arc at point Y.

Step VI

Now, draw $\overrightarrow{PR}$ which is passing through point Y. $m\angle QPR$ is required angle.
Hence,
$m\angle QPR = m\angle AOB = 60°$.



Note While constructing angle of given angle's measurement, only draw step (VI) figure using the above steps and write all steps of construction.

Teaching Point

Explain the concept of angle bisector by using to following activity.

- Divide the class into small groups, and provide each group with a large piece of paper, a protractor, a ruler, a pencil, and some art supplies.
- Instruct each group to make a large X on the paper by connecting the corners of the paper using two diagonal lines.
- Point out the four angles that have formed, and tell students to find and draw the angle bisector of each angle.
- Tell students that this will from additional angles, witch will have their own bisectors, and so on.
- Students should continue finding bisectors until they have a given number of sections.

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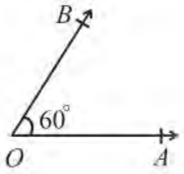
10.3.2 Construction of an Angle Twice in Measure of a Given Angle

Look at the following example to draw an angle twice in measure of a given angle.

Example 7

Construct an angle twice in measure of $m\angle AOB = 60°$

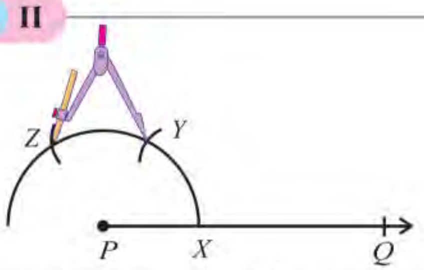


Construction:

Step I

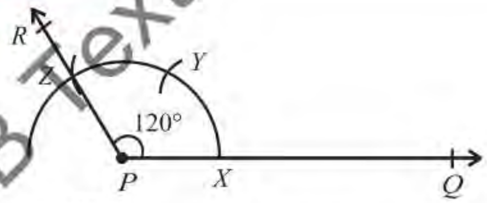
Follow the method which we have used in example-6 Step (I-V) for the construction of $m\angle QPR$.

Step II



Now, draw another arc with centre at point Y having same radius. This arc cut the first arc at point Z, as shown.

Step III



Now, draw $\overleftrightarrow{PR}$ which is passing through the point Z. The angle $m\angle QPR$ is the required angle.

Note

While constructing double angle of given angle's measurement on notebook, only draw step (iii) figure and write all points in construction steps.

10.3.3 Use Compass to Bisect a Given Angle

Bisection of an angle means to divide the given angle into two equal angles. Now look at the following example.

Remember

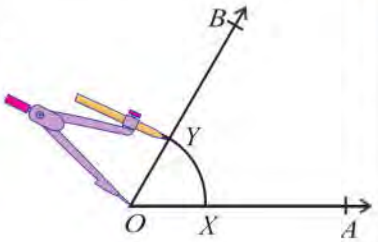
Bisector of an angle means division of given angle into two equal parts.

Example 8 Draw the bisector of the angle $m\angle AOB = 60°$

Construction:

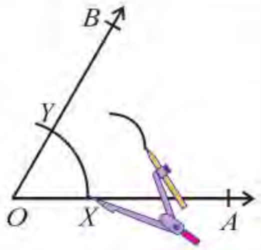
Step I

Draw an arc of any radius at point 'O', which cuts $\overrightarrow{OA}$ at point X and $\overrightarrow{OB}$ at point Y.



Step II

Now, with centre at point X draw an arc of any radius as shown.



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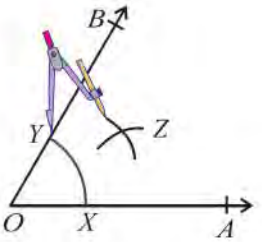
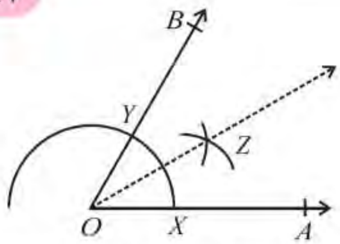
Step III



With centre at point Y draw another arc of same radius which cuts the first arc at point Z.

Step IV

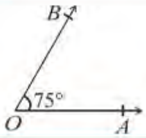


Now, join point O with Z and draw $\overrightarrow{OZ}$. So, $\overrightarrow{OZ}$ is the required angle bisector.

Note While bisecting a given angle on notebook, only draw figure in step (IV) and write all other steps in construction steps.

Practice -6

Divide $\angle AOB$ into two equal angles.



10.3.4 Construction of Angles

The method of construction of different types of angles using compass is illustrated in the following examples.

Remember

An angle of 60° is a basic angle in construction of angles.

Construction of angle 60°

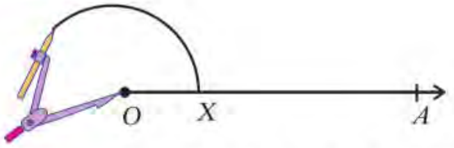
Example 9 Construct an angle of 60° with the help of compass.

Construction:

Step I

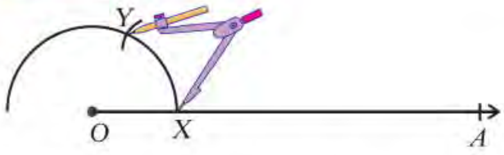
Draw $\overrightarrow{OA}$ of any measurement.

Step II



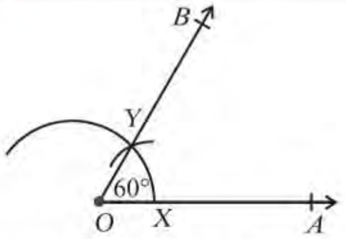
With centre at point O draw an arc of any radius, which cuts $\overrightarrow{OA}$ at point X.

Step III



Now, draw second arc of same radius with centre at point X. Which cuts the first arc at point Y.

Step IV



Join point Y with point O and draw $\overrightarrow{OB}$. Hence, $m\angle AOB = 60°$ is the required angle.

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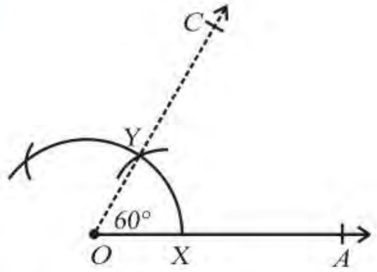
Construction of angle 30°

Example 9 Construct $m\angle AOB = 30°$ using compass.

Construction:

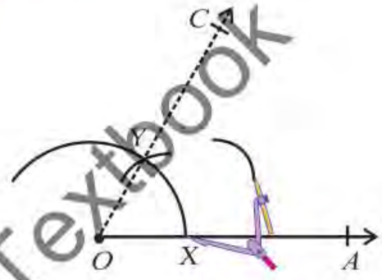
As $\frac{60°}{2} = 30°$, first construct angle $60°$ and then bisect it.

Step I



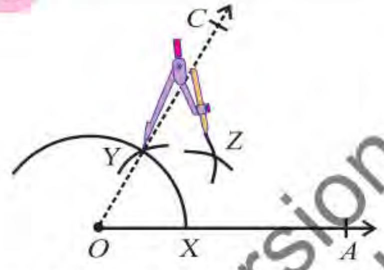
Use all the steps of example 8, and construct $m\angle AOC = 60°$ as shown above.

Step II



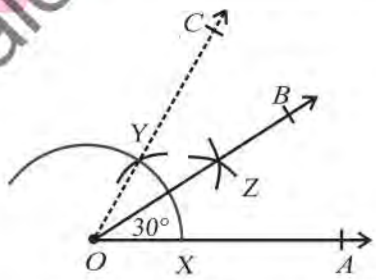
With centre at point X mark an arc of any radius as shown above.

Step III



Now draw third arc with centre at point Y of same radius. The third arc cuts the second arc at point Z.

Step IV



Join point Z with point O, and draw $\overrightarrow{OB}$. Hence, $m\angle AOB = 30°$ is the required angle.

Remember

$m\angle AOB = m\angle BOC = 30°$ and $m\angle AOB + m\angle BOC = 60°$

Construction of angle 90°

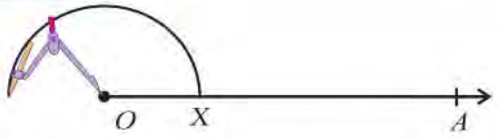
Example 10 Draw $m\angle AOB = 90°$ using compass.

Construction:

Step I

Draw $\overrightarrow{OA}$, as shown above

Step II



Draw an arc of any radius with centre at point O, as shown above. Which cuts $\overline{CA}$ at point X.

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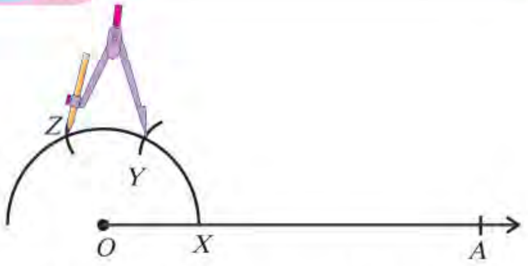
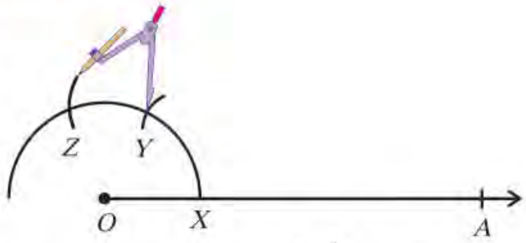
Step III



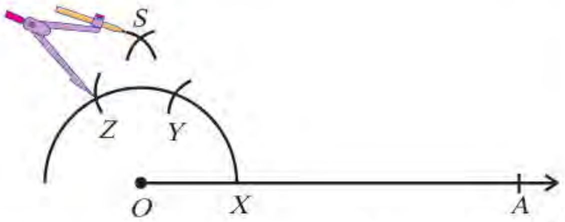
Draw the second arc centre at X cutting the previous arc at Y. Draw third arc centre at point Y cutting the first arc at point Z.

Step IV



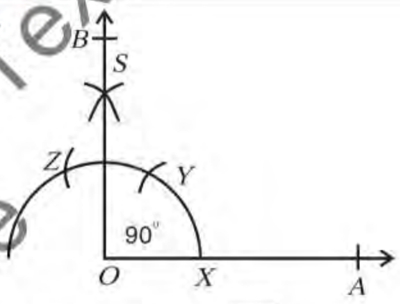
Draw fourth arc of the any radius with centre at point Y.

Step V



Now, with centre at point Z draw fifth arc of the same radius as in step IV which cuts fourth arc at point S, as shown above.

Step VI



Join point S with O and draw $\overrightarrow{OB}$.

Hence, $m\angle AOB = 90°$ is the required angle.

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Construction of angle 45°

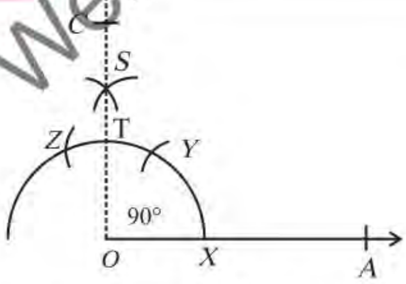
Example 11 Construct $m\angle AOB = 45°$ using compass.

Construction: As, $\frac{90}{2} = 45^{o}$, so construct angle 90^{o} then bisect it.

Thinking Time

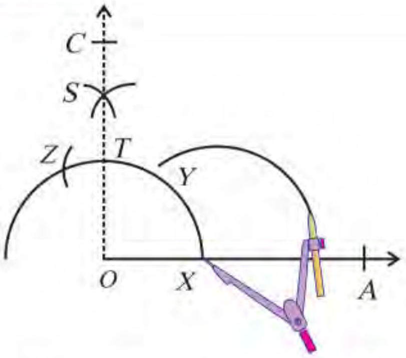
Construct an angle 90° using compass by another method.

Step I



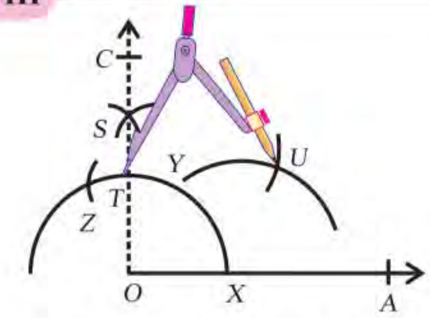
Use all the steps of example 10 and construct an angle of $90°$ where $\overrightarrow{OC}$ passing through the point 'T'.

Step II



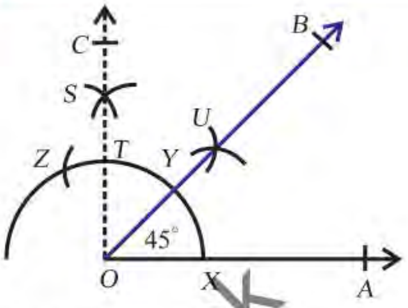
Draw an arc of any radius with centre at point X as shown above.

Step III



Now, with centre at point T draw another arc with previous radius which cuts the first arc at point U.

Step IV



Join point U with O and draw $\overrightarrow{OB}$.

Hence, $m\angle AOB = 45°$ is the required angle.

Thinking Time

Construct an angle 45° using compass by another method.

Remember

$m\angle AOB = m\angle BOC = 45°$ and

$m\angle AOB + m\angle BOC = 90°$

Note

We bisect the angle between 60° and 90°.

We will use the fact $\dfrac{60° + 90°}{2} = 75°$

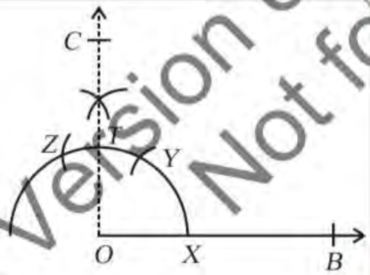
Construction of angle 75°

Example 12 Construct an angle of 75° using compass.

Construction:

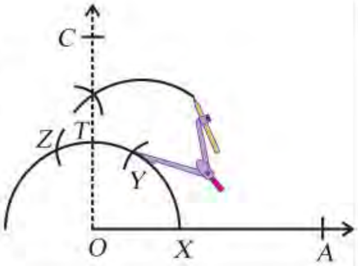
Step I

Use all the steps of example 10 to construct an angle of 90°. Where $\overrightarrow{OC}$ is passing through the point T.

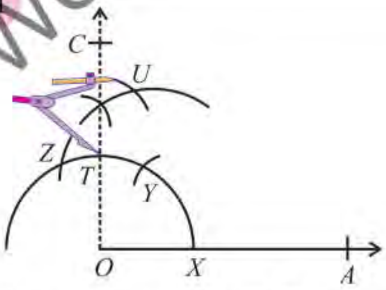


Step II

With the centre at point Y draw an arc of any radius as shown above.

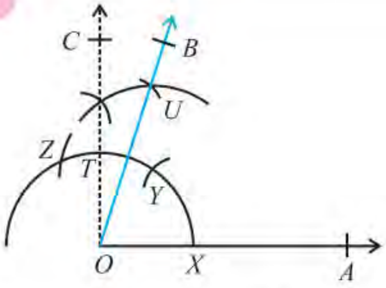


Step III



Now with the centre at point T. Draw another arc with previous radius which cuts the first arc at point U.

Step IV



Join point U with O and draw $\overrightarrow{OB}$.

Hence, $m\angle AOB = 75°$ is the required angle.

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Thinking Time

Draw an angle of 75° with the help of compass by another method if possible.

Remember

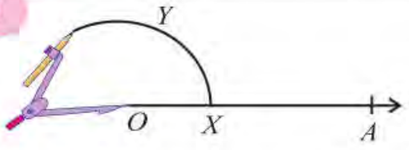
While drawing angle of 75° on notebook, only draw figure of step (IV) and write all steps of construction.

Construction of angle 120°

Example 13 Construct an angle of $m\angle AOB = 120°$ using compass.

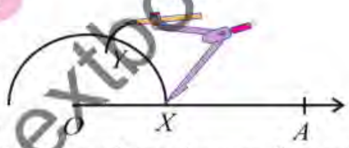
Construction: As, $60° + 60° = 120°$, construct an angle of 60° twice to get an angle 120°.

Step I



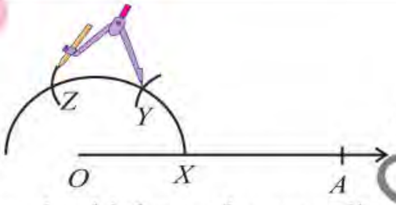
Draw $\overrightarrow{OA}$. Then with centre at point O mark a long arc of any radius which is cuts $\overrightarrow{OA}$ at point X.

Step II



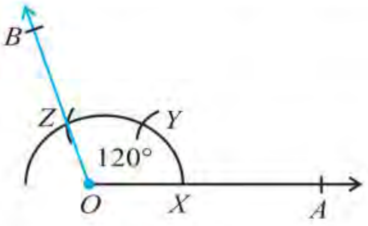
Draw an other arc of previous radius with centre at point X which cuts the first arc at point Y.

Step III



Now, draw the third arc of same radius with centre at point Y, which cuts the first arc at point Z.

Step IV



Join point Z with point O and draw $\overrightarrow{OB}$.

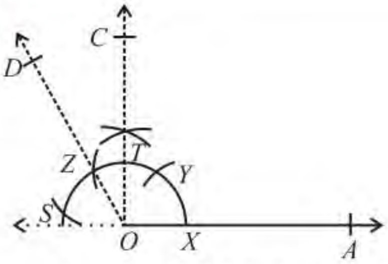
Hence, $m\angle OAB = 120°$ is the required angle.

Construction of angle 105°

Example 14 Construct , $m\angle AOB = 105°$ by using compass.

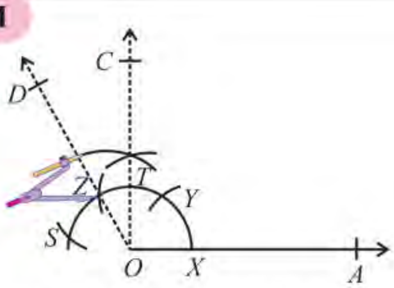
Construction: As $90° + 15° = 105°$, construct an angle of 90° and bisect the angle between 90° and 120°.

Step I



Use all the steps of example 10 and example 13 to construct an angle of 90° and 120° respectively.

Step II



Draw an arc of any radius with centre at point Z as shown above.

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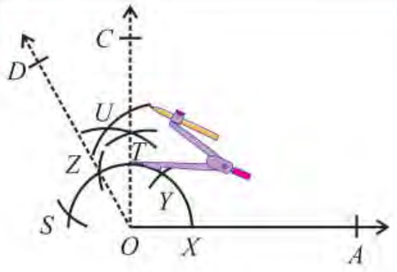
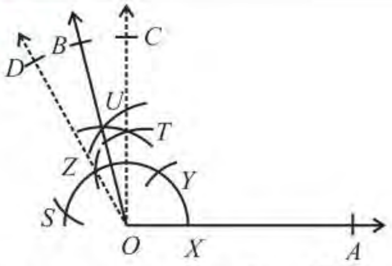
Step III



Now, draw another arc of same radius with centre at point T which cuts the previous arc at U.

Step IV



Join point U with point O and draw $\overrightarrow{OB}$.
Hence, $m\angle AOB = 105°$ is the required angle.

Thinking Time

Draw an angle $105°$ using the equation $120° - 15° = 105°$ if possible.

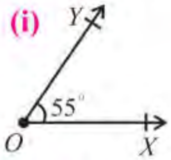
Remember

As, $m\angle AOC = 90°$, $m\angle COB = 15°$
so, $m\angle AOC + m\angle COB = 105°$

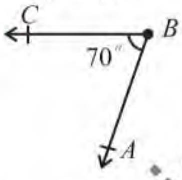
Exercise - 10.2

1. Bisect the following angles:

(i)



(ii)



(iii)

(iv) $90°$ **(v)** $80°$ **(vi)** $120°$

2. Construct the following angles using compass:

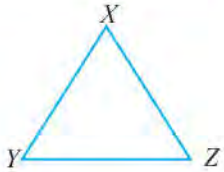
(i) $30°$ **(ii)** $60°$ **(iii)** $120°$ **(iv)** $90°$
(v) $45°$ **(vi)** $75°$ **(vii)** $105°$ **(viii)** $150°$

Practice -1

Use straws to make models of angle 30°, 60°, 90° and 120°.

10.4 FINDING UNKNOWN ANGLES OF A STRAIGHT LINE AND TRIANGLE

In our previous classes, we have learnt about triangles that *"A polygon with three straight edges and three angles is known as triangle".*



In the figure polygon XYZ is a triangle having three sides, $\overline{XY}$, $\overline{YZ}$, $\overline{XZ}$ and three angles $\angle X$, $\angle Y$ and $\angle Z$, respectively. In this section, we will learn different methods to find the unknown angles in a triangle (interior and exterior angles).

Remember

A polygon is a closed shape which has three or more straight edges.

Interesting Information

Samosa is in triangular shape.

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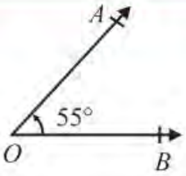
10.4.1 Finding of Unknown Angles

In our previous grades we have learnt about different types of angles. Let's recall these angles. Such as

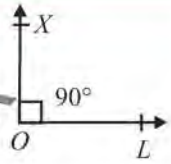
(i) Acute angle

An angle which is less than 90°, is called an acute angle. e.g. $m\angle AOB = 55°$.



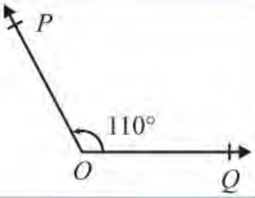
(ii) Right angle

An angle which is equal to 90°, is called right angle. e.g. $m\angle XOL = 90°$.



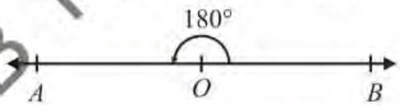
(iii) Obtuse angle

An angle which is greater than 90°, is called an obtuse angle. e.g. $m\angle POQ = 110°$.



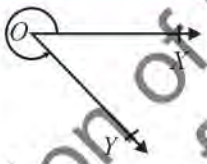
(iv) Straight angle

An angle which is equal to 180°, is called straight angle. e.g. $m\angle AOB = 180°$.



(v) Reflex angle

An angle which is greater than 180°, but less than 360°, is called reflex angle. e.g. $m\angle XOY = 225°$.



(vi) Complete angle

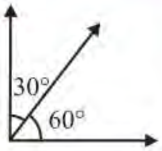
An angle which is equal to 360°, is called complete angle. e.g. $\angle O = 360°$.

We also, need to know about

- Complementary angles
- Supplementary angles
- Adjacent angles

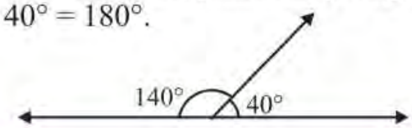
(i) Complementary angles

If the sum of two angles is 90°, then the angles are called complementary angles. e.g. $30° + 60° = 90°$.



(ii) Supplementary angles

If the sum of two angles is 180°, then the angles are called supplementary angles. e.g. $140° + 40° = 180°$.

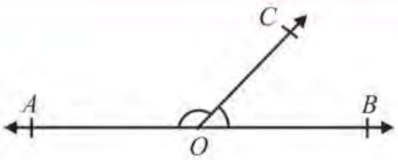


(iii) Adjacent angle

Two angles are said to be adjacent angles if

- they have common vertex
- they have common arm.
- they lie on opposite sides of the common arm.

e.g. $\angle AOC$ and $\angle BOC$ are adjacent angles. $\overline{OC}$ is the common arm and O is the common vertex.



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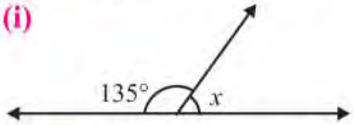
History Lorenzo-Mascheroni was an Italian Mathematician. He was professor of Mathematics at Pavia. He proved that any geometrical construction which is done by compass and straight edge ruler can also be done with the pair of compasses only. To know more about his contribution in Mathematics go online and visit the link.
https://www.mathshistory.st-andrews.ac.uk/Biographies/Mascheroni/

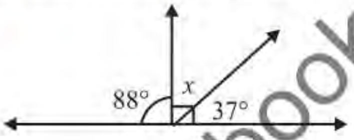
(1750-1800)

Example 15 Find the value of unknown angles in the following figures.

(i)



(ii)



Solution

(i) $135° + x = 180°$ (supplementary angles)
$x = 180° - 135°$
$\therefore x = 45°$

(ii) $80° + x + 37° = 180°$ (supplementary angles)
$x + 117° = 180°$
$x = 180° - 117°$
$\therefore x = 63°$

Practice -9

Find the value of unknown angles in the figure.

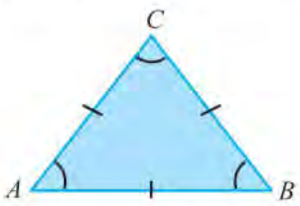
10.4.2 Triangles

We know that *"A triangle is a polygon with three straight edges and three angles"*.
Triangles can be classified into different categories according to these sides and angles.

Types of triangles with respect to sides

(i) Equilateral triangle

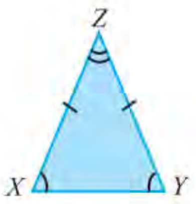
A triangle of which all three sides are equal is called equilateral triangle. e.g ΔABC is an equilateral triangle.



All the angles in an equilateral triangle are 60°.

(ii) Isosceles triangle

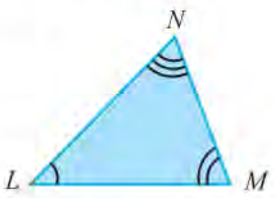
A triangle with 2 equal sides is called isosceles triangle. e.g ΔXYZ is an isosceles triangle.



Two angles of an isosceles triangle are always equal.

(iii) Scalene triangle

A triangle of which all three sides are different, is called scalene triangle. e.g ΔLMN is a scalene triangle.



All angles of scalene triangle are different.

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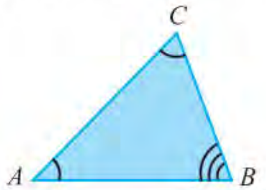
Remember The sum of all interior angles of any triangle is always equal to 180°.

Types of triangle with respect to angles

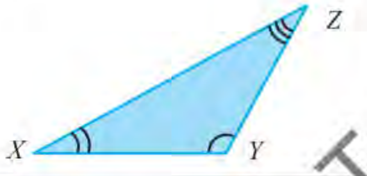
(i) Acute triangle

A triangle, of which all 3 interior angles are acute angles, is called an acute angled triangle e.g ΔABC.



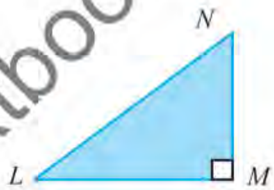
(ii) Obtuse triangle

A triangle, of which 1 interior angle is an obtuse angle is called obtuse angled triangle. e.g ΔXYZ.



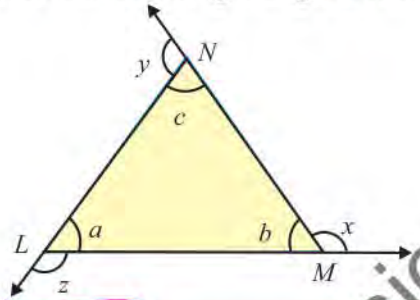
(iii) Right triangle

A triangle, of which 1 interior angle is right angle is called right angled triangle. e.g ΔLMN.



Exterior angles of a triangle

Observe the following triangle *LMN*



(i) $\angle a$, $\angle b$ and $\angle c$ are interior angles of $\angle LMN$.

(ii) $\angle x$, $\angle y$ and $\angle z$ are exterior angles of $\angle LMN$.

"An angle which is formed by any side of a triangle and the extension of its adjecent side is called exterior angle".

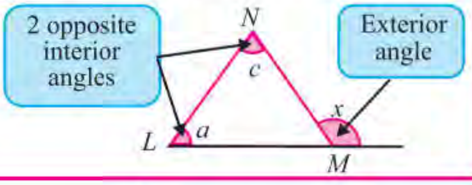
In the above figure $\overline{LM}$ is extended to form $\angle x$. $\overline{MN}$ is extended to form $\angle y$ and $\overline{NL}$ is extended to form $\angle z$.

Fun Fact

(i) An exterior angle is equal to the sum of two opposite interior angles.

(ii) The sum of all exterior angles of any triangle is always equal to 360°. In the above ΔLMN.

$$x + y + z = 360°$$



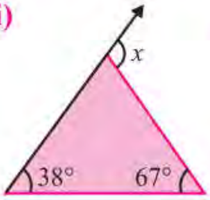
Challenge

Show that the sum of all interior angles of a triangle is equal to 180°.

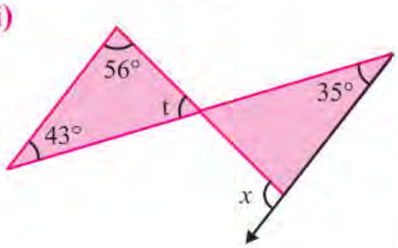
Example 16

Find the value of *x* in the following figures.

(i)



(ii)



Solution

(i) $x = 38° + 67°$ (sum of two apposite interior angles)
$x = 105°$

(ii) $t + 43° + 56° = 180°$ (sum of interior angles of Δ)
$t = 180° - 43 - 56°$
$t = 81°$
$x = 35° + 81°$ (sum of opposite interior angles)
$= 116°$

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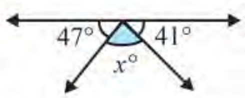
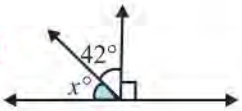
Exercise - 10.3

1. Find the value of unknown angles.

(i)

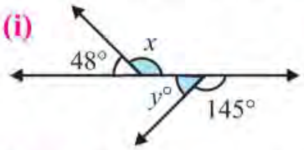
(ii) 

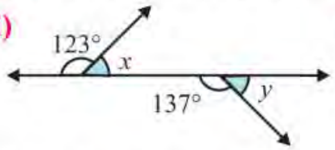
(iii) 

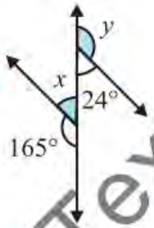
2. The measurement of two angles is 89° and 107°. Do these angles form a straight line?

3. The measurement of four angles is 83°, 142°, 34° and 63°. Which three angles of them form a straight line?

4. Find the value of x and y in the following figures:

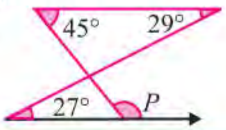
(i) 

(ii) 

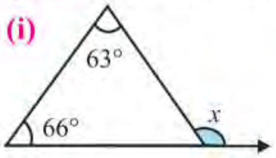
(iii) 

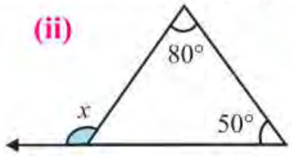
Practice -10

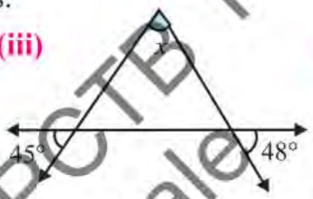
Find the value of *P*.

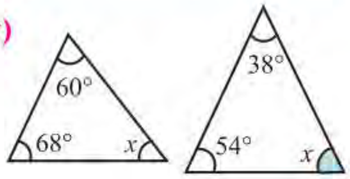


5. Find the value of *x* in the following figures.

(i) 

(ii) 

(iii) 

(iv) 

6. Calculate all the interior and exterior angles of an equilateral triangle.

7. Find the size of each interior angle of regular 3 sides polygon.

Practice -11 Make a star on a chart paper then find the value of each interior and exterior angle of your star. (**Hint**: Use regular polygons in your star).

Review Exercise 10

1. Each of the following questions is followed by four suggested options. In each ease select the correct option.

(i) A line has ______ end points:

a. one b. two c. three d. no

(ii) A ray has _______ starting point/points:

a. one b. two c. three d. none of these

(iii) A line segment has ________ end points:

a. one b. two c. three d. none

(iv) The basic angle in the construction of geometry is:

a. 30° b. 45° c. 60° d. 75°

(v) The angle which can not be constructed using compass is:

a. 45° b. 90° c. 56° d. 60°

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(vi) Any closed shape having three straight edges and three angles is called:

a. pentagon b. hexagon c. triangle d. rectangle

(vii) In the right angled triangle, the largest side is called:

a. base b. perpendicular c. hypotenuse d. line segment

(viii) In equilateral triangle the sum of lengths of any two sides is ____________ the third side:

a. equal to b. less than c. greater than d. none of these

(ix) A right bisector intersects the line at an angle of:

a. 30° b. 60° c. 90° d. 120°

(x) The sum of interior angles in a triangle is always equal to:

a. 60° b. 120° c. 180° d. 270°

(xi) The sum of exterior angles of a triangle is always equal to:

a. 90° b. 270° c. 180° d. 360°

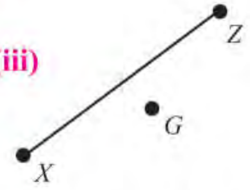
2. Define the following:

(i) Line (ii) Line segment (iii) Ray (iv) Angle (v) Acute angle (vi) Obtuse angle (vii) Right angle (viii) Scalene triangle (ix) Complementary angles (v) Reflex angle

3. Draw the perpendicular to the following lines at given point.

(i)

(ii)

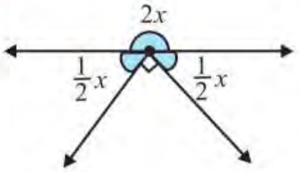
(iii) 

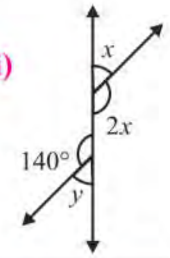
4. Bisect $m\angle A = 80^\circ$.

5. Draw the following angles with the help of compass and then draw the bisector of each angle. (i) 135° (ii) 105° (iii) 45°

6 Divide all the angles into four equal parts, given in Q.5.

7. Find the value of unknown angle in the following figures.

(i) 

(ii) 

Project - 4

Observe the triangles in the structure of the bridge. Search a bridge arround you. Triangles are used in many structures because they are stable. A triangle has three sides and if these are fixed in length, then estimate the angles in the each triangle. Complete the details of your work on a chart paper.

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Summary

- A line is series of points having no end point and it extends in both directions.
- A ray is a line with one end point.
- A line segment is a part of line that is bounded by two distinct end points.
- The right bisector of a line segment is also known as perpendicular bisector.
- The angle 60° is a basic angle in construction of angles.
- Any polygon with three straight edges and three angles is known as triangle.
- A polygon is a closed shape which has three or more straight edges.
- An angle which is less than 90°, is called an acute angle.
- An angle which is equal to 90°, is called right angle.
- An angle which is greater than 90°, is called an obtuse angle.
- An angle which is greater than 180° but less than 360° is called a reflex angle.
- An angle which is equal to 180° is called a straight angle.
- If the sum of two angles is 90° then these angles are called complementary angles.
- If the sum of two angles is 180° then these angles are called supplementary angles.
- Two angles are said to be adjacent angles if
    - they have common vertex
    - they have common arm
    - they lie on opposite sides of the common arm
- A triangle with all sides equal in length, is called equilateral triangle.
- A triangle with two sides equal in length is called an isosceles triangle.
- A triangle of which all three sides are different in length is called scalene triangle.
- A triangle of which all three interior angles are acute angles is called an acute angled triangle.
- A triangle of which 1-interior angle is obtuse angle is called an obtuse angled triangle.
- A triangle of which one interior angle is equal to 90° then the triangle is called right angled triangle.

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Teaching Point

The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

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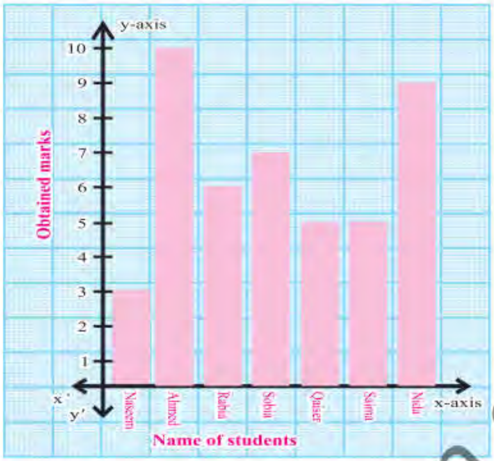
Sub-Domain 11

DATA MANAGEMENT

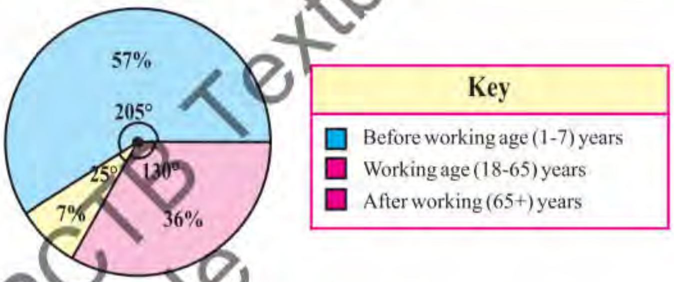
Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Recognize different types of graphs.
- Differentiate between grouped and ungrouped data, continuous and discrete variables.
- Calculate mean, median, and mode.



If each bar in the bar graph represents the obtained marks of the students, then find
- Who has maximum marks?
- Who has minimum marks?



If each sector in the pie graph represents the working stages of population of Pakistan.
- How much part of population represents working age?
- How much part of population represents after working age?

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INTRODUCTION

In our previous classes, we have learnt about data.

"Data is a collection of any information and facts, such as numbers, words, measurements, observations or even description of things."

Numerical data or information is very valuable because it helps us to understand the changes which take place in our daily lives. For example, numerical data or information about temperature helps us to know the condition of water. We can collect the data by several ways i.e. surveys, interviews, observation and questionnaire, etc.

Remember

A survey is a method of collecting information/data about a group.

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11.1 GROUPED AND UNGROUPED DATA

Most surveys or observations produce a large amount of numerical data. It is so complicated to draw the meaningful conclusion from large range of numerical data. To make the meaningful results of data collection, the data is classified into two groups/categories.

Ungrouped data

The data which provides us information or data points individually is known as ungrouped data.

Example 1 The following table shows the scores of Pakistan cricket team in a one day international cricket match. The runs of each player are showing.

Name of players	Sarfaraz	Umar	Hafeez	Shoaib	Yasir	Amir	Kamran	Shahid	Faheem	Waseem	Waqar
Runs	19	25	15	45	80	20	7	18	9	28	5

Grouped data

The data which is given in intervals or groups provide us the information about the data.

In the above example, the runs of the players can be organized into the following classes or groups or intervals.

0 – 19   20 – 39   40 – 59   60 – 79   80 – 99

The below frequency table shows the runs of the players in groups or classes.

Runs	Tally mark	Frequency
0 – 19	𝍸 I	6
20 – 39	III	3
40 – 59	I	1
60 – 79		0
80 – 99	I	1

Activity -1

Ask the obtained marks of class-5 result of your all classfellows and show the result in grouped and ungrouped tables.

11.2 CONTINUOUS AND DISCRETE VARIABLES

Since, we are familiar about variable that a variable is a quantity whose value changes. Here, we will discuss two types of variables.

- Discrete variables
- Continuous variables

Remember The frequency of an event is the number of times the event occurred in an information or study or data.

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(i) Discrete variables

"A variable whose values are obtained by counting is called discrete variable." For example,

a

Blue marbles in the jar.

b

Number of tails when flipping a coin.

c

Number of children in a family.

(ii) Continuous variables

"A variable whose value is obtained by measurement is called continuous variable". For example,

a

Height of a student of grade 6 is 5 feet.

b

Distance from a city to another city is 25.3 *km*.

c

Mass of student is 49.7 *kg*.

Activity -2 Observe in your surrounding and give at least three examples of discrete variables and three of continuous variables.

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Exercise - 11.1

1. Define the following:

- Data
- Ungrouped data
- Grouped data

2. Observe the following tables and identify as grouped or ungrouped frequency table:

(i)

The table shows the ages of students.	
Name of students	Ages in years
Aliya	10
Shumaila	15
Samreen	8
Sameen	7
Aleezay	16
Laiba	11
Arsheena	10

(ii)

The table shows the obtained marks of grade 6 students.	
Marks	Frequency
1 – 20	5
21 – 40	10
41 – 60	12
61 – 80	21
81 – 100	25

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(iii) The below table shows the sales record of toys in a toys shop.

Prices of toys (Rs)	**Tally mark**	Frequency
1 – 100	卌 卌 卌 卌 卌 卌 卌 卌	40
101 – 200	卌 卌 卌 卌	20
201 – 300	卌 卌 卌 卌 卌 卌 卌 卌 卌	45
301 – 400	卌 卌 卌 卌 卌 卌	30
401 – 500	卌 卌 卌 卌 卌 卌 卌	35
501 – 600	卌 卌 卌 卌 卌 卌 卌 III	38

(iv) The table shows the distance covered by a bus in a week.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Distance(km)	150	95	215	317	415	210	125

3. For each of the following, write discrete variable or continuous variable.

(i) Number of people in a school.
(ii) Mass of a mobile.
(iii) Number of pages in your Mathematics book.
(iv) The size of your shoe.
(v) The volume of your water bottle.
(vi) The number of players in football team.

4. A school keeps the record of blood groups of all students who donate blood every year. Is that record a discrete variable or continuous variable?

Practice -1 Count total number of trees and students in your school.

- Arrange your data in the form of grouped or ungrouped data.
- Which type of variable you need to use for your data?

11.3 GRAPHS

Have you seen the graphs around you, like in the newspapers, television, magazines and books etc? The purpose of graph is to show numerical facts in visual form, so that it can be understood quickly, easily and clearly.

In our previous classes, we have learnt about graphs.

"A graph is a diagram representing a system of connections or interrelations among two or more things by a number of lines, pictures, distinctive dots and bars, etc".

We use a graph to display a data in a simple and attractive way. There are different types of graphs such as pictograph, line graph, bar graph and multiple bar graph.

11.3.1 A Bar Graph

"A bar graph is a graphical display of data using bars of different heights."

There are two types of bar graphs: ◆ Horizontal bar graph ◆ Vertical bar graph

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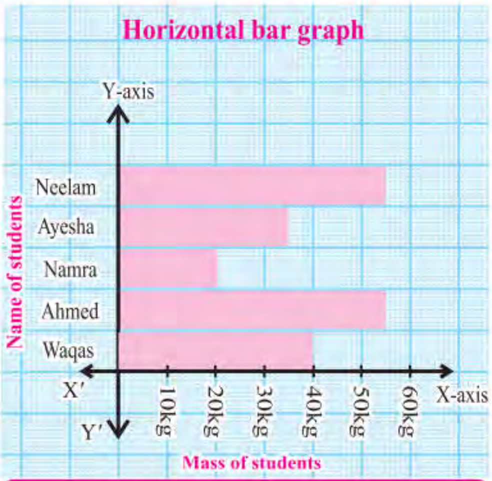
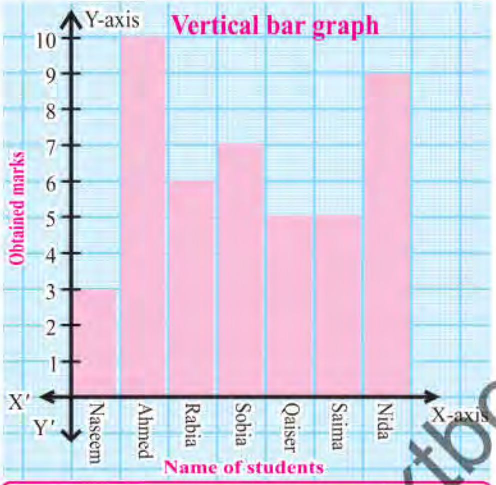


This is bar graph showing the mass of grade 6 students.



This is bar graph showing the obtained marks of students in a class test.

Remember

A bar graph is also known as bar chart.

Note

Always choose a scale that allows all the data to be represented clearly in the available space.

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Simple bar graphs

The procedure to draw the horizontal and vertical bar graphs is explained in the following examples.

Example 2 The students of different classes in a school were asked that how many of students had read the storybooks in summer vacation. The results are shown in the following table.
Show this result in horizontal bar graph.

Name of class	III	IV	V	VI	VII	VIII	IX	X
Number of students	8	3	4	5	9	10	5	6

Solution **Horizontal bar graph for number of students in each class.**

Steps of drawing a horizontal bar graph.

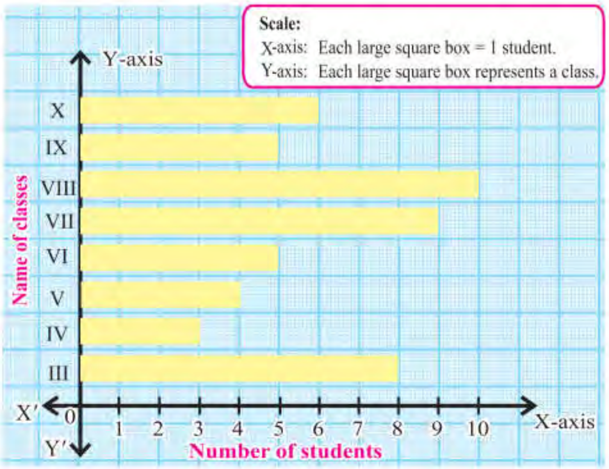
(i) Draw two lines XX′ and YY′ on a graph paper. Both lines are perpendicular to each other and cut at point '0'. The horizontal line is called X – axis and the vertical line is called Y–axis.

(ii) Give the title as Horizontal bar graph for number of students in each graph.

(iii) Lable X– axis as *'Number of students'* and Y– axis as *'Name of classes'*.

(iv) Make a scale, for each axis (on X–axis each large square box = 1 student and on Y–axis large square box represents a class).

(v) Make bars of equal width at equal distance on Y– axis according to the given data.



Example 3 Given below are the seats won by different political parties in the polling outcome of a national assembly election 2018.

Political parties	A	B	C	D	E	F
Seats won	80	55	36	25	10	30

Draw a vertical bar graph to represent the polling results.

Solution **Vertical bar graph for polling results of 2018 election.**

Steps of drawing a vertical bar graph.

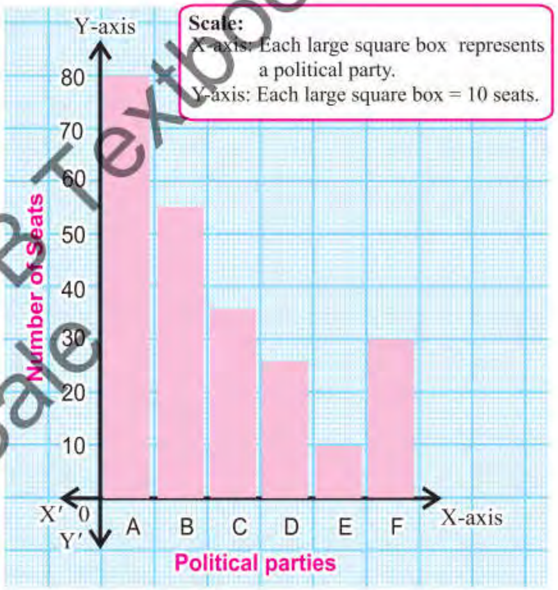
(i) Draw two lines XX′ and YY′ on a graph paper. Both lines are perpendicular to each other and cut at point ' 0'. The horizontal line is called X – *axis* and the vertical line is called Y – *axis*.

(ii) Give the graph title (vertical bar graph for polling results of 2018 election).

(iii) Label X–*axis* as *"political parties"* and Y–*axis* as *"number of seats won"*.

(iv) Make a scale, for each axis (on Y–*axis* each large square box = 10 seats. On X–axis each large square box represents a political party).

(v) Make bars of equal width at equal distance on X–*axis* according to the given information.



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Remember The graphs used in example 2 and example 3 are known as simple bar graphs.

Practice -1 The following table shows the number of copies of a book a publishing house distributed to the dealers in each year from 2015 to 2022.

Years	2015	2016	2017	2018	2019	2020	2021	2022
No. of copies	50,000	40,000	55,000	30,000	60,000	70,000	75,000	90,000

Use bar graph to illustrate the above information.

Multiple bar graphs

"A graph that shows the conformation/data through multiple bars on the graph is called multiple bar graph".

Each data value is represented by a column in the same graph. The procedure to draw the multiple bar graph is illustrated in the following example.

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Example 4 Nadeem is a vegetable shop owner. He recorded the number of kilograms of vegetables sold over two days period. The results are listed on the table below. Create a multiple bar graph to display the data. At the end, list three conclusions that can be made about the data.

Vegetables	1st day sale	2nd day sale
Carrot	35*kg*	40*kg*
Potato	55*kg*	60*kg*
Cauliflower	30*kg*	40*kg*
Peas	50*kg*	40*kg*
Tomatoes	50*kg*	55*kg*
Romaine lettuce	35*kg*	50*kg*

Solution

Steps of drawing a vertical bar graph.

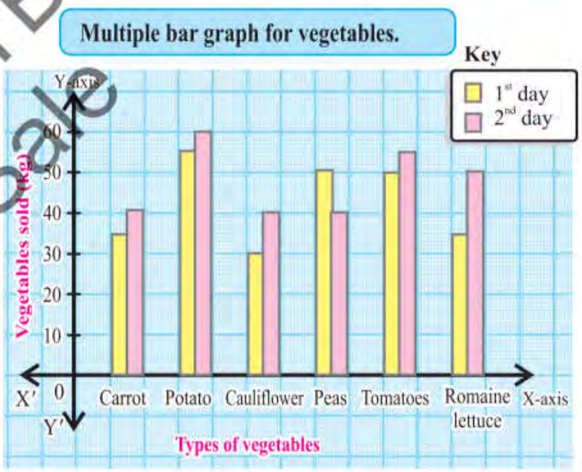
(i) Draw two lines XX′ and YY′ on the graph paper. Both lines are perpendicular to each other and cut at point '0'. The horizontal line is called X-axis and the vertical line is called Y-axis.

(ii) Give title as "multiple bar graph for vegetables."

(iii) Label X-axis as "type of vegetables." and Y-axis as vegetables sold (*kg*).

(iv) Make a key for bars of data.

(v) Make bars of equal width and at equal distance on x-axis.



Example 5 The graph below shows the number of students enrolled in two different schools of Lahore. Read the graph and answer the following questions.

(i) Make a table for enrolled students of both schools.

(ii) How many less students enrolled in school-B in year 2020 than 2019?

(iii) How many total number of students enrolled in school-A in 5 years?

(iv) How many total students enrolled in 2017 in both schools?

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Solution

(i)

Years	2016	2017	2018	2019	2020
School-A	400	700	600	800	200
School-B	500	400	500	600	300

(ii) 600 students – 300 students = 300 students.

(iii) 400 + 700 + 600 + 800 + 200 = 2700

Hence, 2700 students enrolled in school-A in 5 years.

(iv) 700 students + 400 students = 1100 students.

Practice -2 Draw a multiple bar graph for the below set of data that shows the obtained marks of two students.

	Maths	**English**	**Urdu**	**Science**	**Islamiyat**	**GK**
Ali	35	25	28	37	45	48
Ahmed	40	30	45	35	25	40

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Exercise - 11.2

1. The table shows the monthly sale of water coolers in a shop in 2018. Draw the horizontal bar graph.

Name of month	April	May	June	July	August	September
Number of water coolers	50	40	80	35	25	15

2. The following data shows the number of students of school who were late during a week. Construct a horizontal bar graph to show this information.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of students	15	10	25	10	10	27

3. Draw a vertical bar graph for the money collected by a teacher for charity.

Name of students	Ahmed	Humaira	Sobia	Afshan	Aleezey	Rabia	Arslan	Saima
Amount (Rs)	20	90	50	70	30	20	40	10

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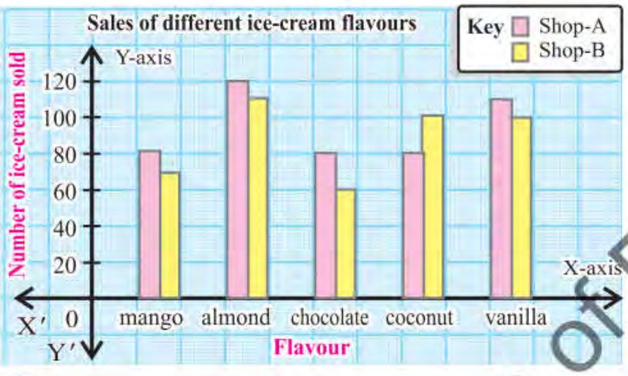
4. Construct vertical bar graph using the daily sale of jeans and dress shirts sale at Ali's shop. The daily sale record is shown in the given table. Also, represent the result on horizontal bar graph.

Day	T – shirts	Dress shirts
Monday	40	20
Tuesday	60	25
Wednesday	35	20
Thursday	80	60
Friday	65	30
Saturday	28	37
Sunday	60	85

5. Draw the multiple bar graph for the data given Q.4.

6. The following bar graph shows the sales record of different flavoured ice-creams of two shopes. Read the graph and give the answers of given questions.



(i) How many mango ice-creams were sold by shop-A?

(ii) Which flavour of ice-cream do people like most in shop-A?

(iii) Find the total number of chocolate ice-creams sold by shop-A and shop-B.

(iv) Which shop sold 80 coconut ice-creams?

Project - 2 Visit your the nearest market and get the latest prices of the following vegetables, show the information on horizontal and vertical bar graphs.

(i)	Potato	Rs._____	**(v)**	Tinda	Rs._____
(ii)	Cauliflower	Rs._____	**(vi)**	Tomato	Rs._____
(iii)	Beet Root	Rs._____	**(vii)**	Okra	Rs._____
(iv)	Carrot	Rs._____	**(viii)**	Bottle Gourd	Rs._____

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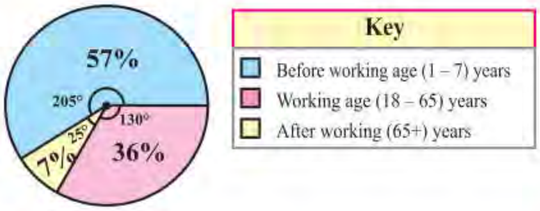
11.3.2 Pie Chart

A graph in which data is represented by "pie chart" or sector of circle is known as pie graph. When we have many quantities in data, we use a pie graph or pie chart to represent these quantities. It shows the contribution of each value to the total.

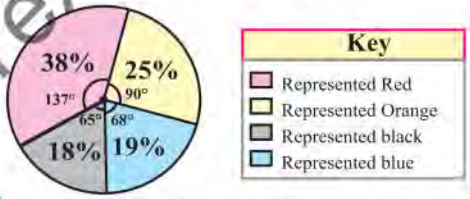


Note This information have been taken in the face.

Reading a pie graph

We know that a circle has an angle of 360°. In a pie chart, the values/quantities are represented by sectors of a circle. It is useful to show or compare various quantities. Look the following example.

Example 6 Waqar did a survey in his school where he asked all the children to name their favourite colours. The results are shown in the pie chart. If there are 800 students in the school, then study the pie graph and answer the following questions:



There are 800 students in all

(i) Which is the most favourite colour a mong the students, and how many students like this colour?

(ii) Which is the least favourite colour among students? Also, tell how many number of students like this colour?

(iii) How many more students like blue colour than black colour?

(iv) How many students like red colour?

Solution Total number of students in the school = 800 students

(i) The most favourite colour among students is orange because its representing sector is the largest in the pie graph.

Orange colour sector represents = 38 %

Number of students like orange colour = 38% of 800

$$= \frac{38}{100} \times 800 = 304 \text{ students}$$

(ii) The least favourite colour among students is black because the representing sector is the smallest in the pie graph.

Black colour represents = 18 %

Number of students like black colour = 18% of 800

$$= \frac{18}{100} \times 800 = 144 \text{ students}$$

(iii) Blue sector represents = 19%

Number of students like blue colour = 19% of 800

$$= \frac{19}{100} \times 800 = 152 \text{ students}$$

Now, Blue sector 's students − black sector 's students

$$152 - 144 = 8 \text{ students}$$

Therefore, 8 more students like blue colour than black colour.

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(iv) Red sector represents = 25%

Number of students like red sector = 25% of 800

$$= \frac{25}{100} \times 800$$

$$= 200 \text{ students}$$

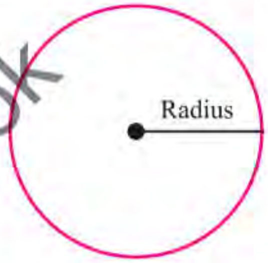
Activity -3

Take a blank sheet and draw a circle with compass and then use information given in example-3 then make a pie chart as given in particular example.

Drawing a pie chart

To draw a pie chart for given data, follow the following steps:

(i) Draw a circle of any radius.



(ii) Find the central angle of each sector by using the formula.

$$\text{Angle of sector} = \frac{\text{Value of the sector}}{\text{Total value}} \times 360°$$

(iii) Join the centre to the circle.

(iv) Start with the horizontal radius and draw radii making corresponding to the value of respective sectors.

(v) Repeat the process for all the data values or groups.

(vi) At the end shade every sector of the data values of different colours.

Example 7 Rabia wrote her daily time table on a paper and wanted to draw a pie chart of her time table. How can she draw the graph? If she spent her time as follow:

Activities	Sleeping	School	Watching T.V	Homework	Playing
Time (hours)	8	6	5	3	2

Solution Total number of hours in a day = 24

Angle of sleeping sector $= \frac{8 \text{ hours}}{24 \text{ hours}} \times 360°$

$= 120°$

Angle of school sector $= \frac{6 \text{ hours}}{24 \text{ hours}} \times 360°$

$= 90°$

Angle of homework sector $= \frac{2 \text{ hours}}{24 \text{ hours}} \times 360°$

$= 30°$

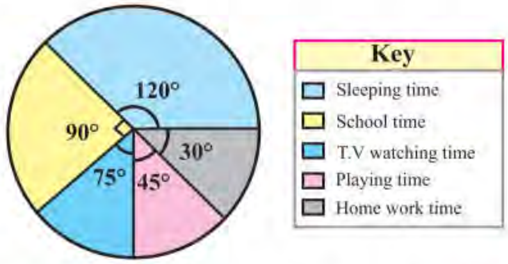
Angle of T.V sector $= \frac{5 \text{ hours}}{24 \text{ hours}} \times 360°$

$= 75°$

Angle of playing sector $= \frac{3 \text{ hours}}{24 \text{ hours}} \times 360°$

$= 45°$

Now, construct pie chart using angle of each sector.



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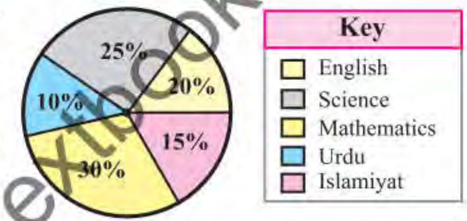
Exercise - 11.3

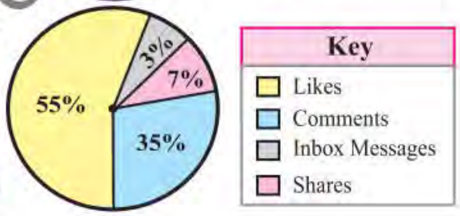
1. Mr. Shahid gave his class surprise tests at the beginning and end of year. The given pie charts shows the grades. Study the pie graphs, do you think he must be happy with the results?

End of term result | **Beginning result**

2. The pie graph shows the percentage of obtained marks. If Waqas obtained 300 marks out of 400 marks, then study the graph and find the marks obtained in each subject. Also, tell in which subjects Waqas obtained maximum and minimum marks?



3. Aleezey is using social media network regularly. The following pie graph is showing the result of likes, comments, inbox massages and shares on her posts in the month of June 2022. Study the pie graph and answer the given questions.



Aleezey received 6000 likes, comments and shares on her posts then.

(i) Find the number of likes, comments, inbox messages and shares on Aleezey's posts.

(ii) How many more likes are there than comments?

(iii) How many less shares are there than likes?

(iv) How many inbox messages are there?

Go Online

Visit the following link to play online game.
https://www.transum.org/software/sw/starter_of_the_day/students/pie_charts.asp

4. Draw the pie charts for the following data tables:

(i) The following table shows the amount of money raised through fund raising by a group of students.

Income source	Income (Rs.)
Charity walk	3000
Donation from parents	2500
Donation from teachers	3200
Ticket selling	8400
Dressing-up-day	2500

(ii) The following table shows the number of students in a school who achieved each grade.

Grades	Number of students
Distinction	100
Merit	250
Pass	400
Fail	100

Project -2 Draw a pie chart of your grade 5 final term obtained marks in each subject.

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11.4 CALCULATION OF MEAN, MEDIAN AND MODE

The most common numerical measures in statistical data are mean, median and mode. These measures are also called averages in statistics. These are also known as measures of central tendency.
"Central tendency of data is the value that represents the whole data or the stages at which the largest number of items tends to concentrate and so it is called central tendency."

(a) Mean

In order to find the mean of few number of quantities, first we add then divide them by the total number of quantities. It can be calculated by formula.

$$\text{Mean} = \frac{\text{Sum of quantities/values}}{\text{Number of quantities/values}}$$

Mean = 47kg

Example 8 The height of 5 children is 1.7 metre, 1.9 metre, 1.4 metre, 1.6 metre and 1.3 metre. Find the mean height of the children.

Solution

$$\text{Mean height} = \frac{\text{Sum of heights}}{\text{Number of children}}$$

$$= \frac{1.7 + 1.9 + 1.4 + 1.6 + 1.3}{5} = \frac{7.9}{5}$$

$$= 1.58\ m$$

Therefore, the mean height of each children is 1.58 *m*.

Example 9 The mean of Rs. 75, Rs. 45, Rs. 60 and x is Rs. 67.5. Calculate the value of x.

Solution Since,

$$\text{Mean value} = \frac{\text{Sum of values}}{\text{Number of values}}$$

$$67.5 = \frac{75 + 45 + 60 + x}{4}$$

$$67.5 = \frac{180 + x}{4}$$

$$67.5 \times 4 = 180 \times x$$

$$270 = 180 + x \quad \Rightarrow \quad x = 270 - 180 = 90$$

Therefore, the required value of x is Rs. 90.

Practice -4

1. In Lahore the temperature of five days in winter was 18°C, 19°C, 20°C, 17°C and 37°C. Find the mean of temperature.
2. The mean of height of five numbers is 10. Find the value of x if the numbers are 79, 94, x, 105 and 200.

Activity -4 Find the mean of listed prices of your all books.

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(b) Median

"The median in statistics is the middle most value in the given set of data."

In order to find the median, we simply arrange the values/data in an ascending or descending order, then find the middle values.

The middle term can be calculated by using the following formula:

$$\text{Middle term} = \left(\frac{n+1}{2}\right)^{th} \text{term, where } n \text{ is total number of observations}$$

Example 10 Find the median of the following set of data 45, 42, 53, 61, 44, 59 and 64.

Solution

$$\text{Middle term} = \frac{n+1}{2} = \frac{7+1}{2} = 4$$

The median is the fourth term in the set of data. So, arrange the data in an ascending order.

Ascending order: 42, 44, 45, 53, 59, 61, 64

53 is the 4^{th} term

Hence, the median is 53.

Example 11 Find the median of 92, 49, 87, 75, 101, 69, 105, 80, 115 and 108.

Solution $\text{Middle term} = \left(\frac{n+1}{2}\right)^{th} \text{term} = \left(\frac{10+1}{2}\right)^{th} \text{term} = 5.5^{th} \text{ term}$

The median is the 5.5^{th} term in the set of data. So, arrange the given set of data in an ascending order.

Ascending order: 49, 69, 75, 80, 87, 92, 101, 105, 108, 115

87, 92 → 5.5^{th} term

$$\text{Median} = \text{Mean of two middle values} = \frac{87+92}{2} = 89.5$$

Hence, the median is 89.5

Remember

If the given set of data is even then find the mean of two middle values to get median.

Practice -5 Find the median of the following set of data:

(i) 7.5, 4.6, 2.9, 3.8, 7.5, 8.4, 5.9 (ii) 9.4, 8.7, 8.5, 7.4, 6.9, 8.9, 7.8, 9.9

Activity -5

Find the mode of all students marks of your class.

(c) Mode

"The value that appears most frequently in the set of data is called mode."

A set of given data may have one mode, more than one mode, or no mode.

In order to find the mode in the given set of data, we simply, observe the data and write the most appeared value.

Example 12 Find the mode of 4, 4, 7, 8, 9, 13, 4, 13, 6, 0, 13, 15 and 13.

Solution Arrange the data in an ascending order. 0, 4, 4, 4, 6, 7, 8, 9, 13, 13, 13, 13, 15

As, the most appeared value is 13. Therefore, mode = 13.

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Practice -6 From the following sizes of kids trousers, find the model size of trousers.
28, 30, 25, 37, 29, 25, 33, 34, 25, 27 and 25.

Exercise - 11.4

1. Find the mean of the following data:

(i) 8, 9, 7, 14, 15, 10 and 20

(ii) 14*kg*, 13*kg*, 10*kg*, 15*kg* and 25*kg*

(iii) 13*m*, 14*m*, 37*m*, 24*m*, 20*m*, 16*m* and 19*m*

(iv) 2.4, 8.4, 7.8, 4.6, 7.7 and 9.7

(v) 134*km*, 415*km*, 168*km*, 250*km*, 424*km*, 342*km* and 241*km*

(vi) 42.6°C, 34°C, 37°C, 41°C, 38.6°C, 42.6°C, and 40.3°C

2. Nadeem read a storybook of 150 pages in 6 days. What is his per day reading of book pages?

3. The marks obtained by the students of grade 6 in the subject of Mathematics are given below.
85, 84, 80, 70, 71, 90, 55, 45, 78, 33, and 39. Find the mean of obtained marks.

4. The mean of 10 numbers is 29.01, seven number of them are 10, 15, 18, 34, 45, 26 and 32. The remaining three numbers are equal to k. What is the value of k?

5. Find the mode and median for each set of data given in Q.1.

6. The scores of 12 students in a Mathematics class test are 18, 17, 18, 13, 9, 11, 13, 24, 22, 18, 11, 18. Find the model score of the class.

7. Find the median of the given set of data 47.5, 40, 25.5, 27.7, 49.8, 32.4, 9.6, 8.4, 40.

Project - 4 Visit your the nearest market. Get the prices of at least five electronic items then find the mean, median and mode of your collected data e.g.

(a)
(b)
(c)
(d)
(e)

Go Online Visit the following link and complete the online quiz about mean, median and mode.
http://www.mathgames.com/skill/7.90-interpret-charts-to-find-mean-median-mode-and-range.

Review Exercise 11

1. Each of the following question is followed by four suggested options. In each case select the correct option.

(i) The collection of any information or fact is called:
a. frequency b. interval c. information d. data

(ii) The data can be classified into:
a. 2 types b. 3 types c. 4 types d. 5 types

Activity -6
Collect the data of ages of 15 people and find the mean, median and mode of your collected data.

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(iii) The tally marks show:

a. data b. class c. frequency d. graph

(iv) The bar graph is also known as:

a. Pie chart b. bar chart c. line graph d. pictograph

(v) Pie slices are used to draw:

a. bar chart b. pie chart c. line graph d. pictograph

(vi) The total angle used in pie chart is:

a. 90° b. 180° c. 270° d. 360°

(vii) The mean is also known as:

a. speed b. distance c. average d. addition

2. Define the following:

(i) data (ii) grouped data (iii) ungrouped data (iv) mean

(iv) median (v) mode (iii) bar graph

3. Which table is showing the grouped data and which is showing the ungrouped data?

(i) The sales record of a month for a bookshop is shown in the table.

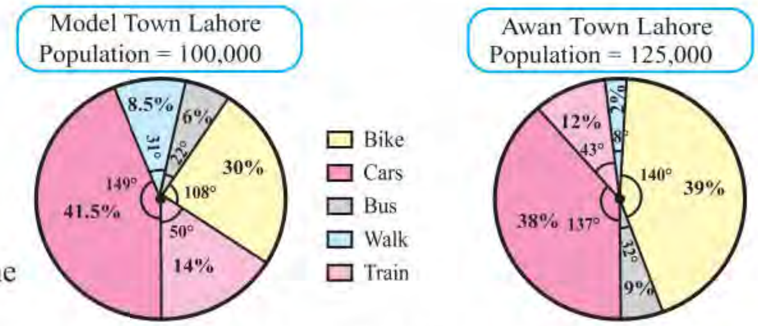
Number of books	Days
201 – 250	4
251 – 300	8
301 – 350	15
351 – 400	5
401 – 450	10

(ii) Number of visitors at an axibition at expo-centre Lahore.

Days	Number of visitors
Saturday	150
Sunday	200
Monday	325
Tuesday	250
Wednesday	175
Thursday	100
Friday	195

4. Draw a horizontal and vertical bar graphs of your obtained marks of each subject of your recent monthly test.

5. These pie charts show how people travel to work in Model Town and Awan Town. The population of the towns are given.



Compare the both graphs and answer the following questions:

(i) In which town do more people travel to work by cars and how many people are travelling by car?

(ii) In which town how many more people are travelling by bikes?

(iii) Find how many people are travelling by walk, train and bus in each town?

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6. The number of students of 7 different classes is given below. Draw the vertical bar graph.

Classes	1	2	3	4	5	6	7
No. of students	15	25	18	29	30	25	35

7. The number of bed sheets manufactured by a factory during six consecutive weeks is given below. Represent this data using bar graph.

Weeks	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
No. of bed sheets	350	200	250	300	150	425

8. The following table shows the record of weekly pocket money of two students. Read the table and draw multiple bar graph.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
First student	35	60	70	90	80	100
Second student	60	80	50	100	55	50

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Summary

- Data is a collection of any information and facts.
- The frequency of an event is the number of times the event occurred in an information/data
- The data which provides us information or data points individually is called as ungrouped data.
- The data which is given in intervals or groups is called grouped data.
- The bar graph is graphical display of data using bars of different height.
- A graph in which data is represented by "pie chart" or sector of circle is known as pie graph.
- In the circle/pie chart the total angle of the circle is 360°.
- Angle of sector can be calculated by using the formula: Angle of sector = $\dfrac{\text{Value of the sector}}{\text{Total value}} \times 360°$
- The average of given data is also known as mean.
- Mean = $\dfrac{\text{Sum of values}}{\text{Number of values}}$
- The middle term in any set of data is called median if it is arranged in an ascending or descending order.
  To calculate middle term, we use the formula: Median = $\left(\dfrac{n+1}{2}\right)^{th}$ term
- The most appeared value in any set of data is called mode.

Teaching Point The questions in the exercises, practices and different activities are given as examples (symbols) for learning. You can use self generated questions (test items) conceptual type MCQ's, fill in the blanks, column matching, constructed response questions and (simple computations) based on cognitive domain (e.g. knowing = 40%, applying = 40% and reasoning = 20%) to assess the understanding of learners.

Sub-Domain 12

PROBABILITY

Students' Learning Outcomes:

By the end of this sub-domain, students will be able to:

- Explain experiments, outcomes, sample space, events, equally likely events and probability of a single event.
- Differentiate the outcomes that are equally likely and not equally likely to occur. (including real- world word problems).

What will happen when you roll a dice once? Predict your possible outcome/ result.

INTRODUCTION

Have you ever heard about a toss before the start of cricket match between two cricket teams (e.g. Pakistan and Indian cricket teams) on television (TV)? A coin is used for toss to decide that which team will bat first.

Similarly, there are many aspects of uncertainty and chances in our routine life. e.g. Will Pakistan cricket team win the match? If throw a dice will I score double of 5? It is likely to rain. Should I go outside with umbrella?

To overcome these types of situations we use the term "probability" in Mathematics.

"Probability is the branch of Mathematics that measures how likely something will happen."

We use it to find the probability of an outcome of an event. Here, we are going to learn a few basic concepts, probability formulas involved to calculate the probability of different types of situations.

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12.1 EXPERIMENT

In statistics "an experiment is any procedure that can be repeated many times and has a well defined sets of possible outcomes". There are two kinds of experiments.

Scientific experiment

This is an experiment for which outcome can be predicted with certainty beforehand such as oxygen and hydrogen or add two numbers such as 4 and 5, etc.

Random experiment

An experiment for which the outcome cannot be predicted except that it is known to be one of the set of possible outcomes, is called a random experiment. e.g. for any given triangle, without knowing the three angles, we can clearly say that the sum of measures of interior angles of a triangle is 180°. Similarly, tossing a coin and rolling a dice, are examples of random experiment.

Experiments			There are 4 green balls and 5 red balls in the bag. A ball is drawn at random.
Possible outcomes			

Since, our interest lies in the random experiment, so in this text by experiment we mean random experiment.

12.2 SAMPLE SPACE AND AN EVENT

> **Note**
> An element of the sample space is called sample point.

Sample space

For a random experiment, the set of all possible outcomes of a random experiment is called a sample space and is denoted by *S*. The elements of *S* are called sample points of outcomes.
For example

(i) Tossing a coin once, then

$S = \{H, T\}$
where H and T are the possible outcomes.

> **Remember**
> **Outcome:** A possible result of a random experiment is called its outcome.

(ii) Tossing a coin twice, then the possible outcomes in the sample space are
$\{HH, HT, TH, TT\}$

(iii) Consider an experiment of rolling a dice. The possible outcomes of this experiment are 1, 2, 3, 4, 5 and 6.
If we are interested in the number of dots on the upper face on the dice.

(iv) Rolling two dice, then we have sample space

$$S = \left\{\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6, 2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix}\right\}$$

> **Practice -1**
> Write down the sample space for tossing a coin three times.

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Events

Let S be the sample space of an experiment. Any subset E of S is called an event associated with the experiment. For example $E = \{HH, TT\}$ is an event associated with the experiment of tossing a coin twice. There are two types of events.

(i) Simple event

"An event which contains a single point of sample space is known as a simple event" e.g.

$S = \{H, T\}$ *then* $E = \{H\}$ is a simple event.

(ii) Compound event

"An event which contains more than one point of sample space is called compound event." e.g.

If $S = \{HH, HT, TH, TT\}$ *then* $E_1 = \{HH, TT\}$ and

$E_2 = \{HT, TH\}$ represent compound events.

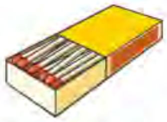
(iii) Equally Likely Events

Two events are said to be equally likely if they have equal chances of happening. For example when we toss a coin there are equal chances of getting a head or a tail. So, getting a head and tail by tossing a coin are equally likely events. When a dice is rolled then getting an odd number and getting an even number are equally likely events.

Note If the random experiment results in P and $P \in S$. We say that the event A occurs or happens.

(iv) Not Equally Likely Events

An event is said to be not equally likely if one or more events are more likely to occur than others.



For example, if you were using a dice that always landed with six facing up, that would be not equally likely event. Similarly, a matchbox has 6 faces but all the faces are not equally likely. So, each face has different occring value.

12.3 PROBABILITY OF SINGLE EVENT

We use a formula to calculate the probability of an event to occur. "How likely something is to occur / happen is called probability".

Let S be the sample space of a random experiment, and E be an event. The probability that an event E will occur, is denoted by $P(E)$. So,

$$\text{Probability of an event occur} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)}$$

Project - 1

Differentiate equally likely and not equally likely outcomes/events with examples from your surrounding.

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Note Not equally likely event is also known as unfair event and equally likely event is also known as fair event.

Practice -2 Shahbaz and Amina has 4 new vehicles this year, 2 cars and 2 wagons. What is the probability that their next number will be a car?

Example 1 Find the Probability of getting an even number when a dice is rolled.

Solution Sample space = $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6 \qquad n(E) = 3$

So, $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{3}{6} = \dfrac{1}{2}$

Example 2 **(i)** Find a probability that a tail will turn up if a coin is tossed.

(ii) Find a probability that an odd number has shown up if a fair dice is tossed.

Solution **(i)** Here $S = \{H, T\}$

Let A = set of tails

$= \{T\}, n(A) = 1$

Since, the outcomes are equally likely, then by using the formula:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

(ii) Here $S = \{1, 2, 3, 4, 5, 6\}$

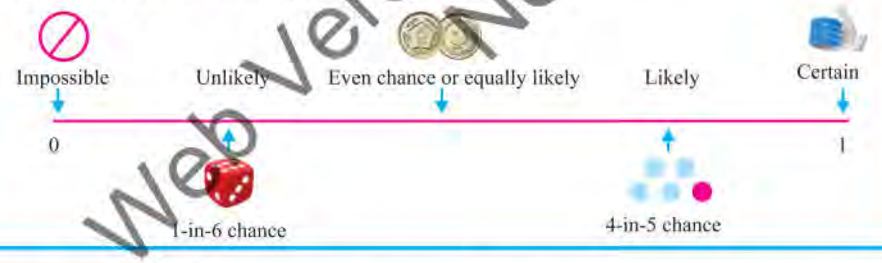
Let B = set of odd numbers has shown up

$= \{1, 3, 5\}$

Since, the outcomes are equally likely then we have

$$P(A) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Remember The probability of an event is always a number from 0 to 1 (inclusive). Look at the below probability scale.



Do you Know?

- If the probability of an event occurring is '0', we call it an impossible event.
- If the probability of an event occurring is '1', we call it certain event.

Example 3 Aleezey is playing cards with her friend when she draws a card from a pack of 56 cards numbered from 1 to 56. What is probability of drawing a number that is perfect of square?

Solution Here $S = \{1, 2, 3, \ldots, 56\}$

B = Set of perfect square numbers.

$= \{1, 4, 9, 16, 25, 36, 49\}$

Since, the outcomes are equally likely then we have $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{7}{56} = \dfrac{1}{8}$

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Exercise - 12.1

1. Each of the letters in the words Jammu Kashmir are on separate cards, face down on the table. If you pick a card at random, what is the probability that its letters will be J and K?

2. What is the probability of rolling a dice at:

(i) rolling a '5'? **(ii)** rolling a number greater than '6'?

(iii) rolling a number less than or equal to '3'?

3. Two unbiased coins are tossed what is the probability of obtaining:

(i) all tails **(ii)** one head **(iii)** one tail **(iv)** all heads

4. There are 5 green colours, 3 red colours and 4 blue colours in a box. What is the probability that:

(i) all colours are green **(ii)** all colours are blue

5. Consider a random experiment of tossing three coins each coin toss only once.

(i) Find the sample space S if we want to determine the exact sequence of tails and heads obtained.

(ii) Find the probability of two tails only.

6. What is the probability of number at least 5 when two dice are rolling once. Also find the probability of number at least 1.

7. There are 200 Covid-19 patients in a private hospital in Lahore. Of these 20 are selected to undergo a special treatment that increases the percentage cured rate from 60% to 85%. What is the probability that a patient received a special treatment if the patient is known to be cured?

8. Two girl students and two boy students enter in a classroom and take 4 seats random in a row. What is the probability that the boy students take the two end seats?

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Review Exercise 12

1. Each of the following questions is followed by four suggested options. In each case select the correct option.

(i) The branch of Mathematics that measures how likely it is that something will happen, is called:

a. event b. probability c. rounding off d. estimation

(ii) There are _______ kinds of expriments:

a. 1 b. 2 c. 3 d. 4

(iii) The set of all possible outcomes of a random experiment is called:

a. sample space b. event c. head d. tail

(iv) A possible result of random experiment is called:

a. event b. estimation c. outcome d. scientific experiment

(v) The sample space for rolling a dice is:

a. $S = \{1, 2, 3, 4, 5, 6\}$ b. $S = \{0, 1, 2, 3\}$ c. $S = \{0, 1, 2, 3, 4\}$ d. $S = \{1, 2, 3, 4, 5\}$

(vi) An event that contains a single point is called:

a. simple event b. compound event c. likely event d. exclusive event

(vii) The probability of single event can be calculated by using the formula:

a. $P(E) = \frac{n(F)}{S}$ b. $P = \frac{E}{S}$ c. $P(E) = \frac{n(E)}{n(S)}$ d. $P(E) = \frac{E}{n(S)}$

(viii) The probability of heads for tossing a coin is:

a. 1 b. 2 c. $\frac{1}{2}$ d. $\frac{1}{3}$

(ix) The probability of letter 'E' in the word MATHEMATICS is:

a. $\frac{2}{11}$ b. $\frac{1}{11}$ c. $\frac{3}{11}$ d. $\frac{1}{2}$

2. Define the following in a short way:

(i) Probability (ii) Experiment (iii) Sample space
(iv) Outcome (v) Simple event (vi) Compound event
(vii) Random experiment (viii) Equally likely event (ix) Scientific experiment

3. The two coins are tossed. Find the probability that two tails are obtained.

4. The two dice are rolled, find the probability that the sum is an even prime.

5. A spinner is divided into 8 equal sectors numbered 1 through 8. If the spinner is spun, is it certain, likely, unlikely, equally likely or impossible that it lands on a number less than 5? Also, find its probability of numbers less than 5.

Summary

- An experiment for which the outcome cannot be predicted except that it is known to be one of a set of possible outcomes is known as random experiment.
- The set of all possible outcomes of a random experiment is known as sample space. It is usually denoted by S.
- Let S be a sample space of an experiment. Any subset E of S is known as an event associated with the experiment.
- Two events are said to be equally likely if they have equal chances of happening.
- An event containing one sample points is called a simple event.
- An event containing at least two sample points is called a compound event.
- Let S be a sample space and E be an event of an experiment then the probability of the event can be calculated by using the formula.

$$\text{Probability of an event occur} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)}$$

Go Online To play the different online games and practice the mathematical concepts visit the following links.

- https://www.educatingyouth.net/videos
- https://www.calculator.net/probability-calculator.html

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ANSWERS

Exercise - 1.1

1. (i) 1, 2, 3, 5, 6, 10, 15, 30 (ii) 1, 41 (iii) 1, 5,13, 65
(iv) 1, 2, 3, 6, 13, 26, 39, 78 (v) 1, 3, 5, 7, 15, 21, 35, 105
(vi) 1, 2, 59, 118 (vii) 1, 3, 5, 11, 15, 33, 165
(viii) 1, 2, 4, 5, 10, 22, 44, 55, 110, 220

2. (i) 7, 14, 21, 28, 35 (ii) 13, 26, 39, 52, 65
(iii) 19, 38, 57, 76, 95 (iv) 43, 86, 129, 172, 215
(v) 59, 118, 177, 236, 295 **3.** 31, 37, 41, 43, 47, 53, 61

4. No even prime numbers **5.** 2

6. 51, 55, 57, 63, 65, 69, 75, 77, 81, 85, 87, 91, 93, 95, 97, 99 **7.** 1

Exercise - 1.2

1. (i) 2, 10; 2, 2, 5 (ii) 3, 35; 3, 5, 7 (iii) 2, 90; 2, 2, 45; 2, 2, 3, 15; 2, 2, 3, 3, 5

2. (i) $5^4 \times 7$ (ii) $2^2 \times 5^4 \times 7^3$ (iii) $3^4 \times 5^3$ (iv) $11^4 \times 13^1 \times 19^3$

3. (i) $2 \times 2 \times 2 \times 2 \times 2$ (ii) $2 \times 3 \times 3 \times 5$
(iii) $2 \times 5 \times 11$ (iv) 5×23 (v) $3 \times 3 \times 5 \times 5$
(vi) $2 \times 2 \times 3 \times 3 \times 3 \times 5$ (vii) $2 \times 3 \times 3 \times 7 \times 11$
(viii) $3 \times 3 \times 5 \times 7 \times 7$ (ix) $3 \times 3 \times 3 \times 5 \times 7 \times 7$
(x) $3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$

4. (i) 2^6 (ii) 5^3 (iii) $2^2 \times 5^2$ (iv) $2^2 \times 5^3$
(v) $2^2 \times 47$ (vi) $2^3 \times 3^4$ (vii) $2^3 \times 3^3 \times 7$
(viii) $2^3 \times 3^4 \times 5$ (ix) $2^3 \times 3^3 \times 5 \times 7$

5. Activity based. **6.** 182 **7.** 70

8. (i) $2\times2\times2\times2\times3\times3\times5\times5\times5$
(ii) $3\times3\times3\times3\times3\times7\times7\times7\times11\times11\times11$
(iii) $5\times5\times5\times5\times5\times5\times7\times7\times7\times11\times11\times11\times11\times11$
(iv) $2\times2\times2\times2\times2\times2\times2\times3\times3\times3\times3\times13\times13\times13\times17\times17$

Exercise - 1.3

1. (i) 3 (ii) 5 (iii) 2 (iv) 5 (v) 14 (vi) 18

2. (i) 10 (ii) 2 (iii) 4 (iv) 32 (v) 6 (vi) 5

3. 1 **4.** No, A number greater than 8 cannot be factor of 8.

5. 2 books of each subject **6.** 10 students **7.** 5*cm*

Exercise - 1.4

1. (i) 72 (ii) 196 (iii) 180 (iv) 1575 (v) 975 (vi) 5250

2. (i) 126 (ii) 225 (iii) 84 (iv) 196 (v) 480480

3. 75 cm **4.** 42 minutes or 2520 sec **5.** 3364

6. 09 : 00 pm or 21 : 00 **7.** 240

Exercise - 1.5

1. (i) 250 (ii) 224 (iii) 2312 (iv) 2695 (v) 25300 (vi) 60800

2. 397120 **3.** 630 **4.** 288

Exercise - 1.6

1. (ii) , (v), (vi), (viii and (x)

2. (i) 169 (ii) 64 (iii) 529 (iv) 289 (v) 961
(vi) 1849 (vii) 9 (viii) 3249 (ix) 5041 (x) 8649

Review Exercise 1

1. (i) a (ii) c (iii) b (iv) b (v) c (vi) a (vii) a
(viii) c (ix) a (x) b

2. 8, 643 **3.** 12 **4.** 12:30 p.m. **5.** 363 **6.** 1:00 p.m.

Exercise - 2.1

1. 5, 6, 7, 8, 9, 10 **2.** 1 **3.** 0, 1, 2, 3, 4, 5, 7, 8, 9

4. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

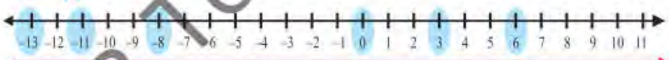
5. (i) Yes. (ii) 0 is whole number but it is not natural number.
(iii) All natural numbers are whole numbers.

6. (i) S = 4, P = 7, A = 12
(ii) A = 54, T = 57, X = 61, R = 65

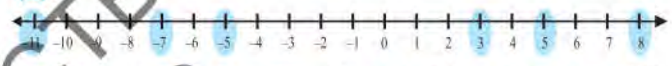
7. (i) –48 (ii) +48

8. (i) – 4 (ii) –2 (iii) I (iv) A (v) C (vi) F

9. (i)



(ii)



10. (i) –14 (ii) +24 (iii) –65 (iv) +39 (v) 0 (vi) +420

11. +12, –17, +26, –57, 0 **12.** 7 **13.** 8

14. 97°F

Exercise - 2.2

1. (i) –7, –4, –2, 0, 1, 3, 4 (ii) –6, –3, –1, 0, 6, 7, 17

2. (i) 16, 7, 0, –1, –2, –4 (ii) 24, 13, 8, 5, 0, –10

3. (i) 45 (ii) 116 (iii) 209 (iv) 0 (v) 2813 (vi) 501

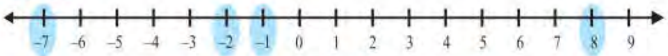
4. (i) 0, 4, 9, 12, 13, 16, 17 (ii) 12, 14, 18, 44, 48, 84

5. (i) 34, 21, 17, 16, 12, 9 (ii) 215, 215, 149, 141, 109, 85, 0

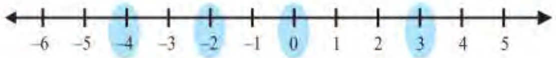
Review Exercise 2

1. (i) d (ii) d (iii) a (iv) a (v) b (vi) b
(vii) a (viii) a (ix) c (x) a (xi) b

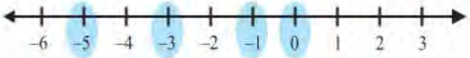
3. (i)



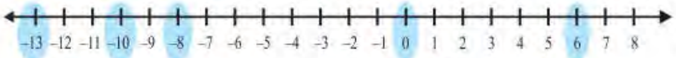
(ii)



(iii)



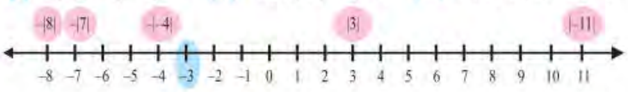
(iv)



4. (i) –4, –3, 0, 2, 4, 7 (ii) –13, –11, –7, 0, 1, 4
(iii) –6, –4, 0, 4, 5, 15

5. (i) 2, 0, –4, –5, –7 (ii) 12, 5, 0, –3, –5 (iii) 8, 5, 3, 0, –2, –6

6.



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7. 1st floor **8.** 15°C **9.** Activity based

10. 15, 0, 18, 15, 13, 215, 131, 54 **11.** 22 *km*

Exercise - 3.1

1. (i) +16 (ii) –10 (iii) –14 (iv) +9 (v) +11 (vi) +39 (vii) +7 (viii) +1 (ix) +15

2. (i) +6 (ii) –50 (iii) +6 (iv) –75 (v) –5 (vi) –97 (vii) +2 (viii) –5 (ix) 29

4. –156 **6.** 62 **7.** 24 *km*

Exercise - 3.2

1. (i) +20 (ii) –24 (iii) 0 (iv) –22 (v) –25 (vi) +30 (vii) 0 (viii) –14

2. (i) –874 (ii) +360 (iii) 285 (iv) –943 (v) 0 (vi) +89

3. (i) –17 (ii) –9 (iii) –6 (iv) +6 (v) –108 (vi) undefined (vii) 0 (viii) –228

4. (i) –8 (ii) –10 (iii) 7 (iv) 720 (v) 0 (vi) 1

9. 222 **10.** Rs.165 **11.** 30°C

Exercise - 3.3

1. (i) –25 (ii) 95 (iii) 186 (iv) 2 (v) 41860 (vi) 230

2. (i) 18.8 (ii) 6.8 (iii) 8.8 (iv) 26.76 (v) -72.96 (vi) 46.8 (vii) 6.3

3. (i) $1\frac{43}{99}$ (ii) $15\frac{13}{135}$ (iii) $2\frac{1}{18}$ (iv) $1\frac{11}{18}$ (v) $1\frac{5}{11}$ (vi) $-1\frac{1}{3}$ (vii) $1\frac{19}{35}$ (viii) $43\frac{5}{12}$

4. Rs. 108,000 **5.** Rs. 87,500 **6.** 7.1 *m*

7. Rs. 16,250 **8.** 2.5 *m* **9.** $7\frac{7}{8}$ metres

Review Exercise 3

1. (i) b (ii) a (iii) b (iv) a (v) b (vi) b (vii) d (viii) b (ix) c (x) b (xi) d (xii) d (xiii) c (xiv) d

3. (i) 74 (ii) –9 (iii) –0.65 (iv) 4.71 **4.** –80 **5.** –18

6. 50 **7.** 585 **8.** –200 **9.** 935 pages

10. 6 hours **11.** 27 *kg* **12.** Rs.180

Exercise - 4.1

1. (i) 80 : 50 (ii) 35 : 55 (iii) 46 : 70 (iv) 85 : 50 (v) 35 : 85 (vi) 55 : 85 : 50 (vii) 46 : 62 : 70

2. (i) 3 : 5 (ii) 7 : 12 (iii) 34 : 37 (iv) 53 : 59 (v) 101 : 103 (vi) $x : y$ (vii) $x : x+1$ (viii) $a - b : p + q$

3. (i) $\frac{5}{6}$ (ii) $\frac{11}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{17}{18}$ (v) $\frac{15}{17}$ (vi) $\frac{17}{19}$ (vii) $\frac{4}{5}$ (viii) $\frac{2}{3}$

4. $\frac{2}{3}$, 2:3 **5.** Expenses = 33000, Savings = 22000 **6.** Moeez: Rs. 26,400, Zurain: Rs. 33,000, Abdul Hadi: Rs. 39,600

Exercise - 4.2

1. (i) Rs. 857 (ii) Rs. 8570 (iii) Rs. 21,425

2. 3 *km/h* **3.** 400 *km* **4.** 6 *m/h* **5.** Rs. 1725

6. (i) Rs. 17.5 (ii) Rs. 1407.5

Exercise - 4.3

1. (i) 33% (ii) 30% (iii) 50%

2. (i) $\frac{1}{4}$ (ii) $\frac{1}{5}$ (iii) $\frac{21}{25}$ (iv) $\frac{17}{50}$ (v) $\frac{93}{100}$

3. (i) 8 (ii) 9 litre (iii) 56 (iv) 720 (v) 92.5 (vi) 270 (vii) 7.84 (viii) 80.1 **4.** 50%

5. 280 students **6.** 525 students **7.** (i) 37.5% (ii) 62.5

8. Rs. 43,750 **9.** Rs. 600 **10.** Rs. 5.43m

11. Rs. 6000 student **12.** march **13.** Aleem **14.** 31.45

15. 9 **16.** Rs. 51,750 **17.** Rs. 7320.32 **18.** 17.65%

Exercise - 4.4

1. (i) 19:45 (ii) 21:31 (iii) 20:35

2. (i) 03:48 p.m. (ii) 11:15 p.m. (iii) 07:43 p.m.

3. 10:30 p.m. **4.** 04:45 **5.** 30 minutes

Exercise - 4.5

1. (i) $3\frac{8}{9}$ *m/sec* (ii) $11\frac{1}{4}$ *m/sec* (iii) $69\frac{4}{9}$ *m/sec*

2. (i) 90 *km/h* (ii) 468 *km/h* (iii) $1225\frac{4}{9}$ *km/h*

3. 107 *km/h* **4.** 160 *km* **5.** yes **6.** 84.29 *km/h*

Review Exercise 4

1. (i) b (ii) c (iii) a (iv) b (v) d (vi) b (vii) b (viii) c (ix) b (x) b (xi) d (xii) b (xiii) c

3. (i) 9:10 (ii) 1:3 (iii) 13:21 (iv) 7:8:9 (v) 2:3:4 (vi) 4 to 5 to 6

4. Rabia: Rs. 3948, Saima: Rs. 6580, Alina: Rs. 9212

5. 333.3 words **6.** 35% **7.** 240 votes **8.** Rs. 1950

9. 160 **10.** Bananas = 345, Apples = 390 **11.** 60 boxes **12.** yes

13. Rs. 90,000 **14.** 12 hours

Exercise - 5.1

1. (i) It is a set because all geometrical instruments are distinct
(ii) It is a set because all vehicles are distinct
(iii) It is not a set because tea cups are repeating
(iv) It is a set because all flowers are distinct

2. (i) False (ii) False (iii) True (iv) True (v) Ture (vi) True (vii) False (viii) False (ix) True

3. (i) $\in$ (ii) $\notin$ (iii) $\in$ (iv) $\notin$ (v) $\notin$ (vi) $\in$ (vii) $\notin$ (viii) $\in$ (ix) $\in$

4. (i) A = {spring, summer, autumn, winter}
(ii) B = {Wednesday} (iii) Activity based (iv) Activity based
(v) E = {p, u, n, j, a, b} (vi) F = {Rajab, Shaban, Ramadan, Shawwal, Dhu al Qidha, Zhul Hajj}
(vii) G = {Imran Khan, Nawaz Sharif, Shahid Khaqan Abbasi, Yousaf Raza Ghaliani}

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(viii) H = {2, 3, 5, 7} (ix) I = {a, e, i, o}

5. (i) A = Set of first five whole number
(ii) B = Set of four games
(iii) C = Set of English alphabets
(iv) D = Set of all provincial capitals of Pakistan
(v) E = Set of last six months of solar colander
(vi) F = Set of whole numbers between 20 and 31
(vii) G = Set of all provinces of Pakistan
(viii) H = Set of even numbers upto 14
(ix) I = Set of three fruits

Exercise - 5.2

1. (i) empty set (ii) empty set (iii) not an empty set
(iv) empty set

2. (i) finite (ii) finite (iii) finite (iv) infinite (v) finite
(vi) finite (vii) finite (viii) finite (ix) infinite

3. (i) n(A) = 3, not singleton (ii) n(B) = 1, singleton
(iii) n(C) = 5, not singleton (iv) n(D) = 1, singleton
(v) n(E) = 5, not singleton (vi) n(F) = 1, singleton

4. (i) $\supset$ (ii) $\longleftrightarrow, \subseteq$ (ii) $\subset$ (iv) $\longleftrightarrow$ (v) $\supset$
(vi) $\subset$ (vii) $\supset$ (viii) $\supset$

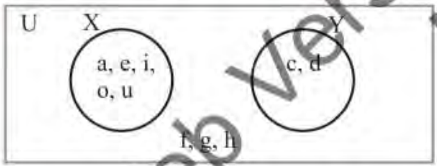
5. (i) equal (ii) equal (iii) equal (iv) equal
(v) equivalent (vi) equivalent

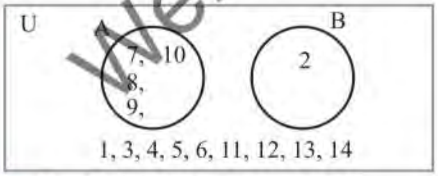
6. **Proper subsets:** { }, {Red}, {Yellow}, {Blue}, {Red, Yellow}, {Red, Blue}, {yellow, Blue}
Improper Subsets: {Red, Yellow, Blue}

8. n(A) = 6 **9.** Activity based

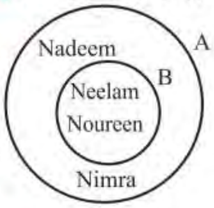
Exercise - 5.3

1. U = {goat, cat, hen, dog}
A = {hen, cat, dog}

2.


3.


4. (i) A = {1, 2, 3, 4, 5, 6}, B = {1, 3, 5}
(ii) U = {2, 4, 5, 6, 7, 8, 9, 10, 13}, A = {7, 13}, B
(iii) X = {1, 3, 5, 7, 9}, Y = { }

5.


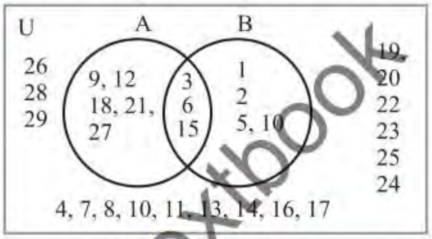
Review Exercise 5

1. (i) a (ii) b (iii) a (iv) c (v) b
(vi) d (vii) c (viii) b (ix) a (x) x

3. (i) Activity based (ii) B = Set of negative whole numbers
(iii) {school diary}

4. { }, {1}, {2}, {1, 2}, {8}, {1, 8}, {2, 8}, {1, 2, 8}

5. U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12},
A = {1, 2, 3, 5, 7}, B = {2, 4, 6, 8, 9}

6.


Exercise - 6.1

1. (i) 21,25 (ii) 307,317 (iii) 42,49 (iv) –78, –76

2. (i) 25, 36, 49 (ii) 83, 81, 79 (iii) 24, 28, 32 (iv) 13, 21, 34

Exercise - 6.2

1. (i) $n + 6 = 20$ (ii) $n - 27 = 5$ (iii) $4n < 7$ (iv) $2n > 25$

2. (i) False (ii) True (iii) True (iv) False (v) False

3. (i) 18 (ii) 6 (iii) 400 (iv) $\frac{3}{10}$ (v) $\frac{15}{22}$

4. (i) $C = 9, 0$; $V = n$ (ii) $C = 7, 2, 9$, $V = p$
(iii) $C = \frac{1}{3}$, $V = xy$ (iv) $C = 7, 4, 9$, $V = i$
(v) $C = 8$, $V = m, n$ (vi) $C = 9, -37$, $V = x, y$

Exercise - 6.3

1. (i) $3a + 8$ (ii) $5p - 5$ (iii) $5a \div 8b$
(iv) $4\,ny$ or $n\ (4y)$ (v) Rs. $170n$

2. (i) $2n$ (ii) n (iii) $n - 3$

3. (i) n ; 3 (ii) $2n$; $6m$ (iii) ℓmn (iv) $4xy, 6y, -5$ (v) ab,2

4. (i) 4,3 (ii) 23 (iii) $\frac{2}{5}$ (iv) 1,85 (v) 15, 81

5.

	Constant	Variable	Coefficient
i	1	a	8
ii	–4	a, b	3, 2
iii	0	p, q	1
iv	C	xy	a, b
v	C	a, b	3, 4

6. $3a, 5a, 9a, 76a$
$21m, 4m$
$8p, 3p$
$2s$
8ℓ

7. (i) $x + y$ (ii) $6a$
(iii) $4x - y$ (iv) $p + pq + q + r$
(v) $12a^2 + 3b^2$ (vi) $\frac{1}{y} + \frac{2}{x} + \frac{3}{2}$
(vii) $2xy + yz + 4zx$
(viii) $9x + 6x^2$

8. $(25g + 27b)$, $(23g + 30b)$, $48g + 57b$ **9.** $75b, 53f, 114c$

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Exercise - 6.4

1. (i) $5x$ (ii) $11p+13q$ (iii) $15a$ (iv) $30xy$
(v) $98pq+2qr+23rs$ (vi) $15xy+6pq+54$ (vii) $13x+10y+5z$

2. (i) $11p-q$ (ii) $\frac{6}{7}a+\frac{1}{3}b$ (iii) $15xy+11yz$
(iv) $38x+11y-8z$ (v) $13p+14q-r+10$ (vi) $5q+13r$

3. (i) $12a-23b$ (ii) $13xy-7z$ (iii) $9x-5y+4z$
(iv) $-\frac{2}{3}\ell-\frac{2}{5}m+14$ (v) $-49\ell-11m+8n$

4. (i) $-x+5y-12z$ (ii) $2(3x-3yz)$ (iii) $6m+2n-2$
(iv) $13p-4q+7r$

5. $p+2q+3r$ 6. $8x+8y+8z+8$ 7. $3x+7y+5z$

Exercise - 6.5

1. (i) $8x+3y$ (ii) a (iii) $3a+42\frac{1}{3}b$ (iv) $4x+7a$
(v) a + b (vi) $4x-11y+2z$

2. 8b+ (12b – 3b)+(20b – 2b), Total bags of sweets = 35 bags.

3. (i) -5 (ii) -76 (iii) $\frac{85}{14}$ (iv) 62
(v) $\frac{7}{12}$ (vi) $\frac{9}{8}$ 4. -336 5. 29

Review Exercise 6

1. (i) c (ii) b (iii) b (iv) a (v) a (vi) a
(vii) b (viii) d (ix) a (x) c

3. (i) $23x-6y-4z$ (ii) $22xy+2zy+zx$

4. (i) $2x+y+1$ (ii) $-2x-4y-2z+3$

5.

	Coefficient	Variable	Constant
a	a, b	p, q	c
b	$2.5, \frac{1}{3}$	x,y	14
c	13, q	p, q	5

6. (i) $17p+2q$ (ii) $4xy+7yz+4zx-30$ 7. $12x-4y+8z$

8. (i) $7p+q-10r+45s$ (ii) $-14x-8y+12$ (iii) $5x-y-8$

9. 24 10. 36, 49

Exercise - 7.1

1. (i) $4(9)+4=40$ (ii) $2(4)-2=6$ (iii) $2n-3=20$
(iv) $\frac{1}{2}x+6=22$ (v) $3(6)\div3=6$ (vi) $x+7=14$

3. {i, ii, iv, vii, xi} are expressions
{iii, v, vi, viii, ix and x} are equations

Exercise - 7.2

1. i, iii, iv are linear equation

2. (i) $n+8=18$ (ii) $y-7=14$
(iii) $(x+1)+(x+2)=29$ (iv) $4n=2n+23$

3. (i) 12 (ii) 7 (iii) 5 (iv) 35 (v) 15 (vi) $15\frac{2}{3}$

4. 4 5. 5 6. 20 7. 9 8. 90

9. 37 10. 30 years 11. 80 and 120 12. 43, 560 kg

Review Exercise 7

1. (i) b (ii) b (iii) a (iv) a (v) b (vi) a
(vii) c (viii) a (ix) a (x) b

2. (i) -15 (ii) 4 (iii) 10 (iv) $4\frac{1}{2}$ (v) -3.5 (vi) $-\frac{41}{560}$

3. (i) -10 (ii) 0.98 (iii) $1\frac{7}{20}$ (iv) 1 (v) $\frac{3}{4}$ (vi) 9

4. 13 5. 47 years 6. 60 and 40

Exercise - 8.1

1. (i) 14.8 cm (ii) 13 cm (iii) 16 cm (iv) 14 cm

2. (i) A = 18cm^2 , P = 16 cm (ii) A = 16 cm^2 , P = 16 cm
(iii) A = 14 cm^2, P = 18 cm (iv) A = 5 cm^2 , P = 12 cm

3. A = 45000 mm, P = 860 mm 4. 12.25 m 5. 30 cm, 32 cm

6. 32 cm 7. 36 km^2 8. Rs. 14400 9. Rs. 11000

10. (i) 96 cm^2 (ii) 41 cm^2, 1 cm^2 (iii) 24 cm^2

11. Rs. 163500 12. 264 m^2

Exercise - 8.2

1. (i) $\overline{BC}$ (ii) $\overline{RS}$ and $\overline{PT}$ (iii) $\overline{AF}$ and $\overline{CE}$

2. (i) 24 cm^2 (ii) 50 cm^2 (iii) 18 cm^2 (iv) 25 cm^2
(v) 12 cm^2 (vi) 25 cm^2

3. (i) 3.2 cm^2 (ii) 57 cm^2 (iii) 49 cm^2

4. 14 cm 5. 11 cm 6. 7 cm 7. 55 cm 8. 49500 flowers

Exercise - 8.3

1. (i) 60 cm^3, 96 cm^2 (ii) 729 cm^3, 486 cm^2 (iii) 5145 mm^3, 8470 mm^2

2. (i) 352 cm^2 , 448 cm^3 (ii) 9.5 cm^2 , 580 cm^3
(iii) 3 cm , 948 cm^3 (iv) 0.5 cm , 56 cm^3
(v) 9 cm , 990 cm^2 (vi) 17 m, 4913 m^3
(vii) 11 m, 726 cm^2

3. 25000 m^3 4. 3375 5. 2400 cm^2 6. 90 m^3

7. 598.5 m^3 8. 1248 cm^2 9. Rs. 3800

Review Exercise 8

1. (i) c (ii) c (iii) c (iv) b (v) a (vi) d (vii) b
(viii) b (ix) c (x) b (xi) c (xii) c (xiii) b
(xiv) b (xv) a (xvi) a (xvii) c (xviii) a (xix) a

3. (i) 16 cm^2 (ii) 17 cm^2 (iii) 12 cm^2

4. (i) 34 cm, 30 cm^2 (ii) 33 cm

5. Momina 6. Rs. 121,500 7. 25 cm^2

8. Rs. 60,000 9. 180 cm^3 10. 70 sheets

11. 510 square feet

Exercise - 9.1

1. (i) Cuboid (ii) Cone (iii) Sphere (iv) Cube
(v) Cylinder (vi) Hemisphere

2.

1-D	2-D	3-D
Line	Square, rectangle triangle	Cube, cuboid cylinder

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3. (i) Surfaces = 3
Vertices = 0
Edges = 2
(ii) Surfaces = 6
Vertices = 8
Edges = 12
(iii) Surfaces = 6
Vertices = 8
Edges = 12
(iv) Surfaces = 1
Vertices = 1
Edges = 1
(v) Surfaces = 1
Vertices = 0
Edges = 1

Exercise - 9.2

1. (i) Parallel lines (ii) Intersecting lines
(iii) Intersecting lines (iv) Parallel lines
(v) Intersecting lines (vi) Intersecting lines
2. (i) $a = 30°, b = 150°, c = 150°$ (ii) $r = 120°, p = q = 60°$
3. (i) d and f, b and h, a and g, c and e, m and i, ℓ and o, n and j, k and p
(ii) d and e, a and h, m and k, n and ℓ
(iii) f and b, c and g, i and o, j and p
4. (i) $x = 80°$ (ii) $x = 120°$ (iii) $x = 23$
(iv) $x = 22.5$ (v) $x = 80°$

Exercise - 9.3
Activity based

Review Exercise 9

1. (i) c (ii) b (iii) b (iv) d (v) c
(vi) c (vii) b (viii) d (ix) b (x) d
4. (i) $x = 34°$ (ii) 247° 5. Activity based

Exercise - 10.1
Activity based

Exercise - 10.2
Activity based

Exercise - 10.3

1. (i) 54° (ii) 92° (iii) 48° 2. No 3. 83°, 63° and 34°
4. (i) x = 132°, y = 35° (ii) x = 57°, y = 43°
(iii) x = 15°, y = 156°
5. (i) 129° (ii) 130° (iii) 87° (iv) 68°
6. Each interior angle=60°, Each exterior angle=120° 7. 60°

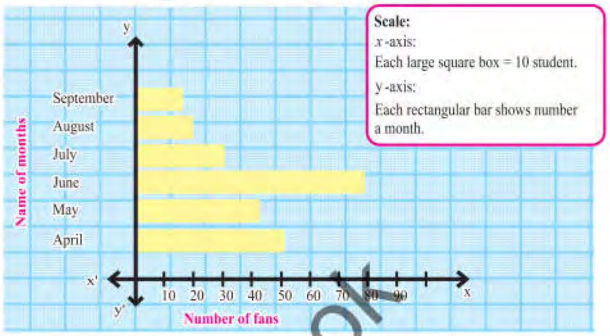
Review Exercise 10

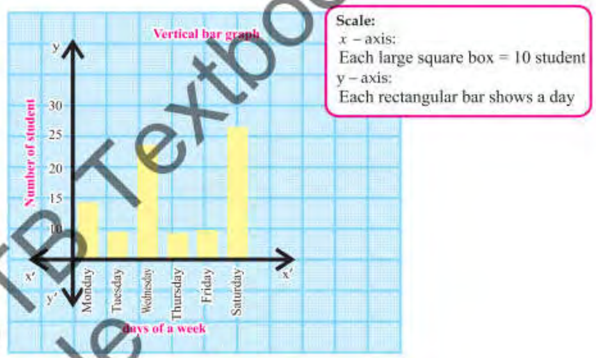
1. (i) d (ii) a (iii) b (iv) c (v) c (vi) c
(vii) c (viii) c (ix) c (x) c (xi) c
7. (i) 90° (ii) $x = 60°, y = 40°$

Exercise - 11.1

2. (i) Ungrouped (ii) Grouped (iii) Grouped
(iv) Ungrouped
3. (i) Discrete variable (ii) Continuous variable
(iii) Discrete variable (iv) Continuous variable
(v) Continuous variable (iii) Discrete variable
4. Continuous

Exercise - 11.2

1.


2.


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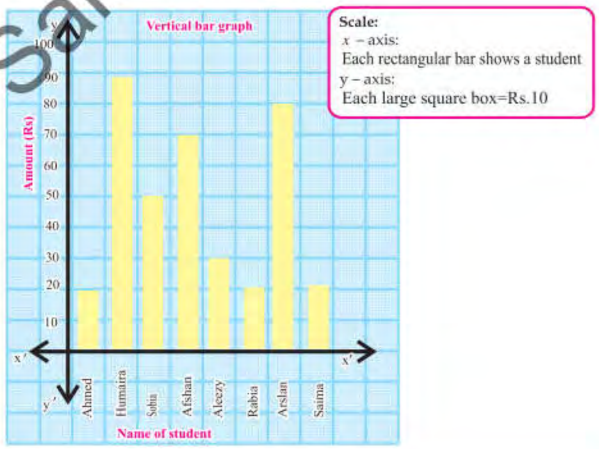

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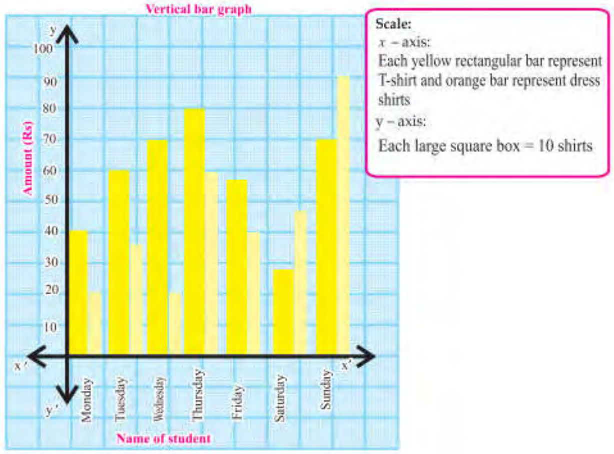

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6. (i) 80 (ii) almond (iii) 140 (iv) A

Exercise - 11.3

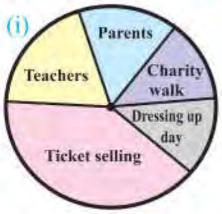
1. He must be happy because the end of term result is better

2. English: 75 Science: 60 Mathematics: 90
Urdu: 30 Islamiat: 45

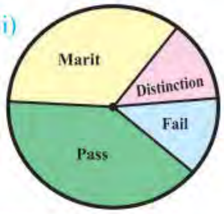
3. (i) likes: 3, 300 , comments: 2100 ,
Inbox message: 420 , shares: 180

(ii) 1200 (iii) 3,120 (iv) 420

4. (i)



(ii)



Exercise - 11.4

1. (i) 11.86 (ii) 15.4 *kg* (iii) 20.4 *m* (iv) 6.8
(v) 282 *km* (vi) 39.4°C

2. 25 pages **3.** 66.4 marks **4.** $k = 36.7$

5.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Mode	no mode	no mode	no mode	no mode	no mode	42.6
Median	10	14	19	7.75	250	40.3

6. 18 **7.** 30.05

Review Exercise 11

1. (i) d (ii) a (iii) c (iv) b (v) b
(vi) b (vii) d (viii) c (ix) d

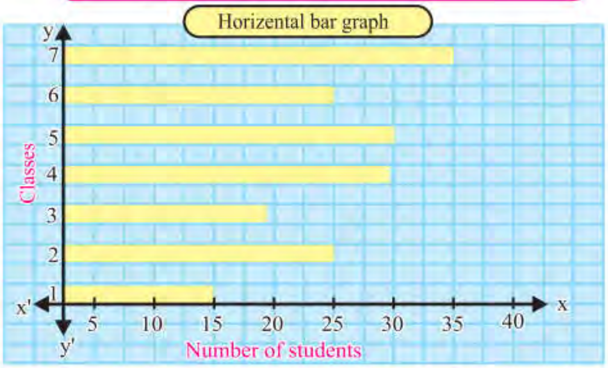
3. (i) grouped data (ii) Ungrouped data **4.** Activity based

5. (i) Model town , 41, 500 people (ii) 18, 750 more people

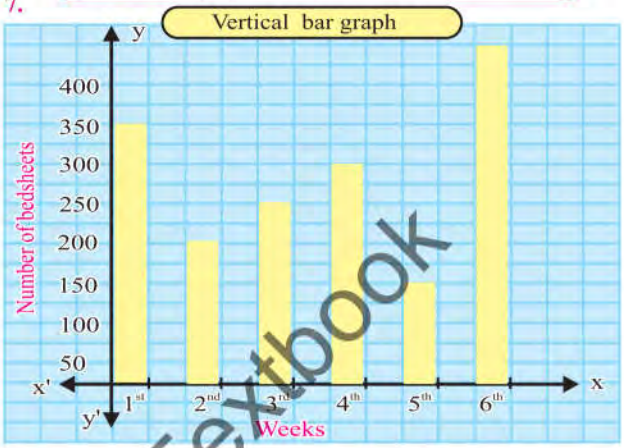
(iii)

Name of town	Walk	Train	Bus
Model town	6,000	14, 000	8,500
Awan town	2,500	11, 250	15, 000

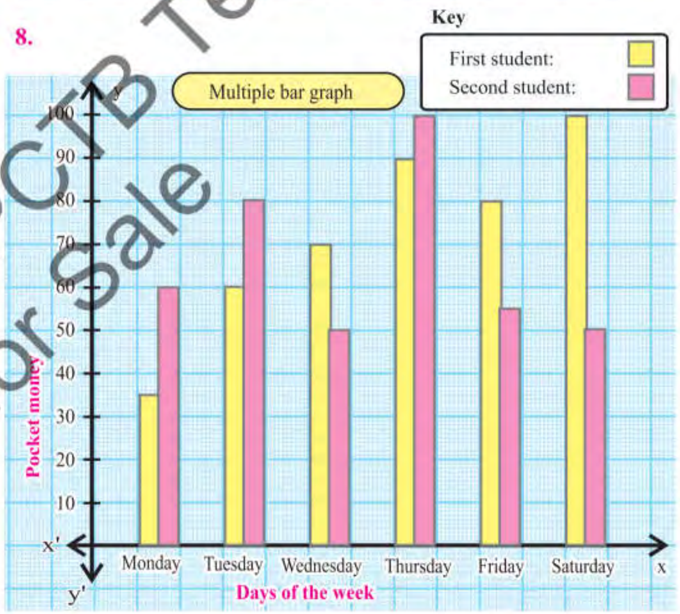
6. Scale: x-axis: Each large square box represent 5 students.
y-axis: Each green bar represent a class.



7. Scale:
x-axis: Each orange bar represents the weeks.
y-axis: Each large square box represents 50 bedsheets.



8.



Exercise - 12.1

1. (i) $\frac{1}{5}$ **2.** (i) $\frac{1}{6}$ (ii) 0 (iii) $\frac{1}{12}$

3. (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$ (iv) $\frac{1}{4}$ **4.** (i) $\frac{1}{12}$ (ii) $\frac{1}{12}$

5. (i) {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} (ii) $\frac{3}{8}$

6. $\frac{11}{36}$ and $\frac{11}{36}$ **7.** $\frac{459}{10000}$ or 0459 **8.** $\frac{1}{2}$ **9.** $\frac{35}{143}$

Review Exercise 12

1. (i) b (ii) b (iii) a (iv) c (v) a
(vi) a (vii) c (viii) c (ix) c (x) b

3. $\frac{1}{4}$ **4.** $\frac{1}{36}$ **5.** equally likely $\frac{1}{2}$

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GLOSSARY

Absolute Value: The positive distance from zero to any number on the number line is called absolute value.

Algebraic Equation: An algebraic equation is an open mathematical sentence which contains an equal sign i.e

Area: The size of surface or amount of surface covered by any closed shape is called area.

Altitude: The altitude is measure of the shortest distance between the base and its top.

Bar Graph: A bar graph is a graphical display of data using of different heights.

BODMAS: BODMAS, stands for brackets, order of operation, multiplication division, addition and subtraction.

Cube: A 3–dimensional solid body bounded by six square surfaces is known as cube.

Cuboid: A 3 – dimensional solid body bounded by six rectangular surfaces is known as cuboid.

Cylinder: A 3–dimensional solid body with two circular surfaces of equal radius and one curved surface between them is known as cylinder.

Cone: A 3–dimensional solid body with one circular surface and a slanting surface is known as cone.

Data: Data is a collection of any information or facts.

Empty Set: A set which does not contain any element is called empty set.

Even Numbers: Even number is a number which is multiple of 2.

Factor: A factor is a number which divide the dividend completely leaving no remainder.

Grouped Data: The data which is given in intervals / groups, it provides us the information about groups it is known as grouped data.

Graph: A graph is a diagram representing a system of interrelation among two or more things by a number of lines, pictures distinctive dots and bars etc.

Set: A set is a collection of well defined and distinct objects.

Finite Set: A set having a finite number of elements is known as finite set.

Infinite Set: A set having unlimited number of elements is known as finite set.

Singleton Set: A set which contains only one element is called singleton set.

Subset: A subset is a set which contain in another set.

Superset: A superset is a set which contain another set.

Subset: A proper subset of a set A is subset of A which is not equal to set A.

Improper Subset: Improper subset is a subset which contains all the elements of original set.

Natural Numbers: Natural numbers are counting numbers. 1, 2, 3, …, are all natural numbers.

Continued Ratio: The comparison among three or more quantities is called continued ratio.

Rate: A rate is a special kind of ratio in which the two terms are in different units.

Universal set: A universal set is a collection of all elements or members of all related sets known as subsets of universal set.

Venn diagram: A diagram that represent the possible logical relationship between finite collection of sets pictorially.

Pattern: A pattern is a some phenomenon that repeats regularly based on a set rule or condition.

Parallel lines: Two or more lines which extends in the same directions and remain the same distance apart.

Intersecting lines: Two lines that are intersecting each other at a single point is called intersecting lines.

Point of intersection: The point where two lines are intersecting each other.

Perpendicular lines: Two or more lines that meet or intersect each other at right angle.

Transversal: A line that passes through two or more of the parallel lines.

Vertically opposite angles: When two lines intersect at a point then opposite angles formed because of intersection.

Symmetry: A line that divide an object into two identical pieces.

Reflective symmetry: If a shape or pattern is reflected in a mirror line or line of symmetry then this type of symmetry is called reflective symmetry.

Rotational symmetry: If a shape is rotated about a point to another position and still look the same then the shape has rotational symmetry.

Order of symmetry: Number of times a shape looks the same in one full turn is called order of symmetry.

Centre of rotation: A find point around which the rotation occurs is called the centre of rotation.

Acute angle: An angle which is less than 90°.

Right angle: An angle which is equal to 90°.

Obtuse angle: An angle which is greater than 90° but less than 180°.

Straight angle: An angle which is equal to 180°.

Reflex angle: An angle which is greater than 180° but less than 360°.

Complete angle: An angle which is equal to 360°.

Complementary angles: If the sum of two angles is equal to 90°, then angles are complementary angles.

Supplementary angle: If the sum of two angles is 180°, then the angles are supplementary angles.

Equilateral triangle: A triangle with all three sides are equal.

Isosceles triangle: A triangle with 2 equal sides.

Scalene triangle: A triangle with all three sides are different.

Acute triangle: A triangle with all three interior angles are acute angles.

Obtuse triangle: A triangle with one interior angle is obtuse angle.

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Mathematical Symbols / Notatoins

Symbol	Name	Symbol	Name
$=$	equal to	$\times$	multiplication
$\neq$	not equal to	$\div$	division
$>$	greater than	$\mid\;\mid$	absolute value
$<$	less than	$\Rightarrow$	meterial implication
$\leq$	less than or equal to	$\forall$	for all
$\geq$	greater than or equal to	$\{\;\}$	set brackets
$+$	addition	ϕ	empty set
$-$	subtraction	$\in$	belongs to
$\subseteq$	subset	$\notin$	does not belong to
$\subset$	proper subset	$(\;)$	parenthese
$\supset$	superset	$\parallel$	parallel
$\perp$	perpendicular	$\approx$	approximately equal to
$\%$	percentage	π	pi
$\angle$	angle	$\cup$	universal set
$:$	ratio		

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- Mathematics 6 published by PTB published in 2018.
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